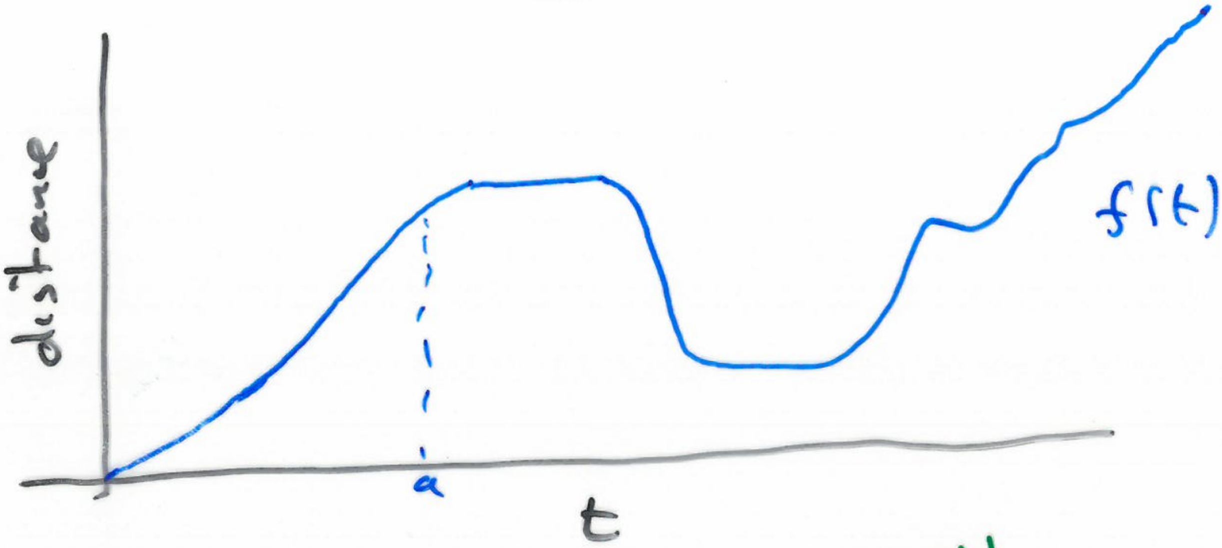
Second homework now online

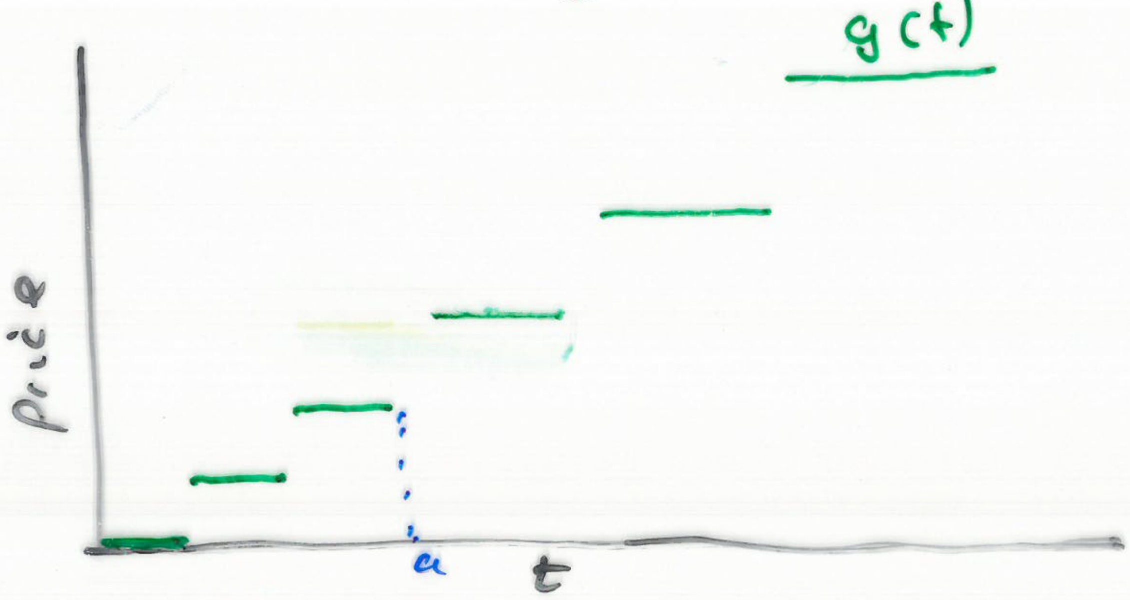
Continuity

I travel to Dublin airport.

$f(t)$ = distance from Galway t minutes after leaving.

$g(t)$ = price of parking my car t minutes after entering the car park.





Intuitively: continuous means there are no breaks in the graph.

Better, more general definition:

A function $f(t)$ is continuous if a small change in the input yields only a small change in the output.

In the above example $f(t)$ is continuous, whereas $g(t)$ is not.

In the context of functions $f: \mathbb{R} \to \mathbb{R}$, the following is the best definition.

We say that $f(t)$ is continuous at a point $t = a$ if:

i) $f(a)$ is defined. (i.e. a lies in the domain of f.)

ii) $\lim_{t \to a} f(t)$ exists, and

iii) $\lim_{t \to a} f(t) = f(a)$.

<u>Example</u> Determine the constant k such that

$$f(x) = \begin{cases} x^3 & \text{for } x \geq 2 \\ kx & \text{for } x < 2 \end{cases}$$

is continuous at all points x.

Soln The only problem is

at $x =$

We need

$$\lim_{x \to 2} f(x) = f(2) = 8$$

i.e. we need

$$\lim_{x \to 2-} f(x) = 8 = \lim_{x \to 2+} f(x)$$

i.e. need

$$\lim_{x \to 2-} kx = 8 = \lim_{x \to 2+} x^3$$

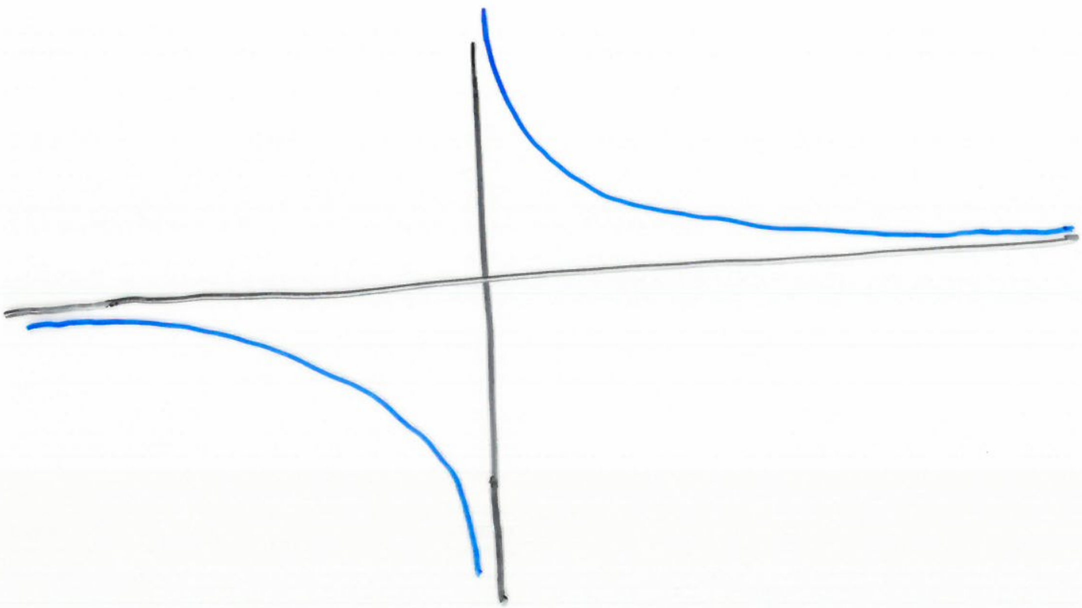
So we need

$$2k = 8$$

So we need $k = 4$.

Defn A function f is said to be **continuous** if it is continuous at all points x in the domain of f.

Example Is the function $f(x) = \frac{1}{x}$ continuous?



Yes!

Intermediate Value Theorem

Suppose

$$y = f(x)$$

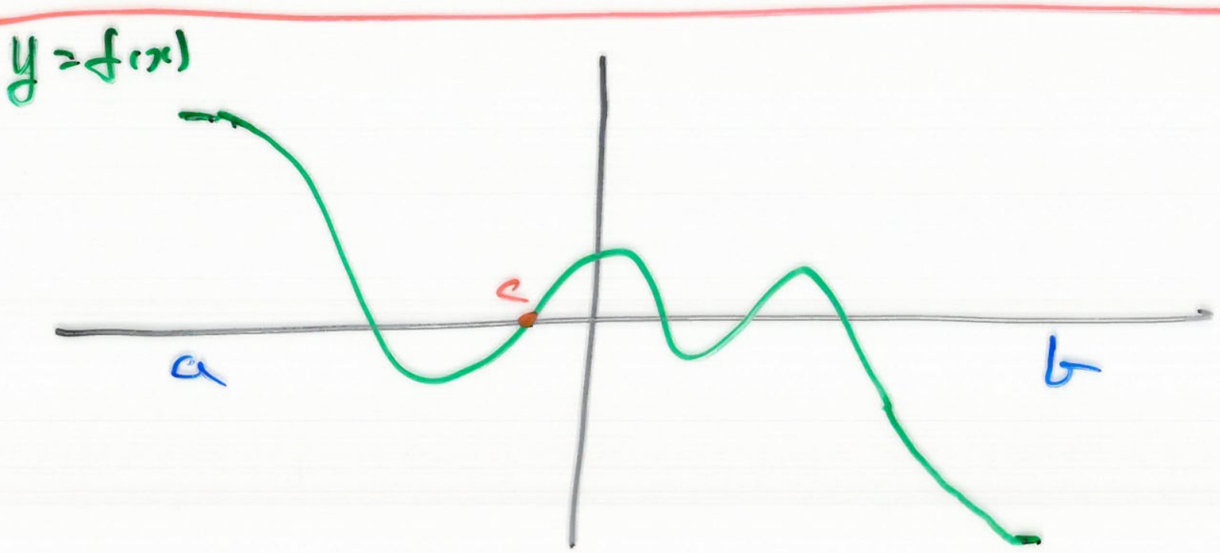
is continuous at all points x in the range $a \le x \le b$.

Suppose also that

$$f(a)\, f(b) \le 0.$$

Then there exists at least one value c in the range $a \le c \le b$ such that

$$f(c) = 0.$$



Example Prove that

$$x^3 - x - 1 = 0 \qquad (*)$$

has a solution in the range $1 \le x \le 2$.

Sol^n

Let $f(x) = x^3 - x - 1$.

"clearly" this function $f(x)$ is continuous.

$a = 1$, $b = 2$.

$$\left.\begin{array}{l} f(1) < 0 \\ f(2) > 0 \end{array}\right\} \text{ so } f(1)f(2) < 0.$$

The IVT says that there is at least one value c, $1 \le c \le 2$, such that

$$f(c) = 0.$$

i.e. c is a solution to $(*)$.