Recap of yesterday:

$$\lim_{x \to 1} \frac{x^6 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x^5 + x^4 + x^3 + x^2 + x + 1)}{\cancel{(x-1)}}$$

$$= 6.$$

The formal definition of limit can be used to prove the following, not too surprising, result.

Proposition

Suppose

$\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$

both exist. Then

i) $\lim_{x \to a} (f(x) + g(x)) =$

$$\lim_{x \to a} f(x) + \lim_{x \to a} g(x).$$

ii) For any $k \in \mathbb{R}$

$$\lim_{x \to a} k f(x) = k \lim_{x \to a} f(x).$$

iii) $\lim_{x \to a} (f(x)\, g(x)) =$

$$= \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$$

iv) $$\lim_{x \to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

provided $\lim_{x \to a} g(x) \neq 0$.

Example

$$\lim_{x \to 2} \frac{x^2 + 4x + 5}{x^2 + 5}$$

$$\overset{(iv)}{=} \frac{\lim_{x \to 2} (x^2 + 4x + 5)}{\lim_{x \to 2} (x^2 + 5)}$$

$$\overset{(i)}{=} \frac{\lim_{x \to 2} x^2 + \lim_{x \to 2} 4x + \lim_{x \to 2} 5}{\lim_{x \to 2} x^2 + \lim_{x \to 2} 5}$$

$$\overset{(ii)\,\&\,(iii)}{=} \frac{\left(\lim_{x \to 2} x\right)^2 + 4 \lim_{x \to 2} x + \lim_{x \to 2} 5}{\left(\lim_{x \to 2} x\right)^2 + \lim_{x \to 2} 5}$$

$$= \frac{2^2 + 4.2 + 5}{2^2 + 5}$$

$$= \frac{17}{9}.$$

x - intercept : point where the graph of $y = f(x)$ intersects the x - axis, $(f(x) = 0)$

y - intercept : point where graph of $y = f(x)$ intersects the y - axis.

Example

$f(x) = x^2 - 1$.

Domain of $f = \mathbb{R}$

Codomain of $f = \mathbb{R}$

$f : \mathbb{R} \longrightarrow \mathbb{R}$.

Range of $f = \{ f(x) \in \mathbb{R} : x \in \text{Domain} \}$

$= [-1, \infty)$

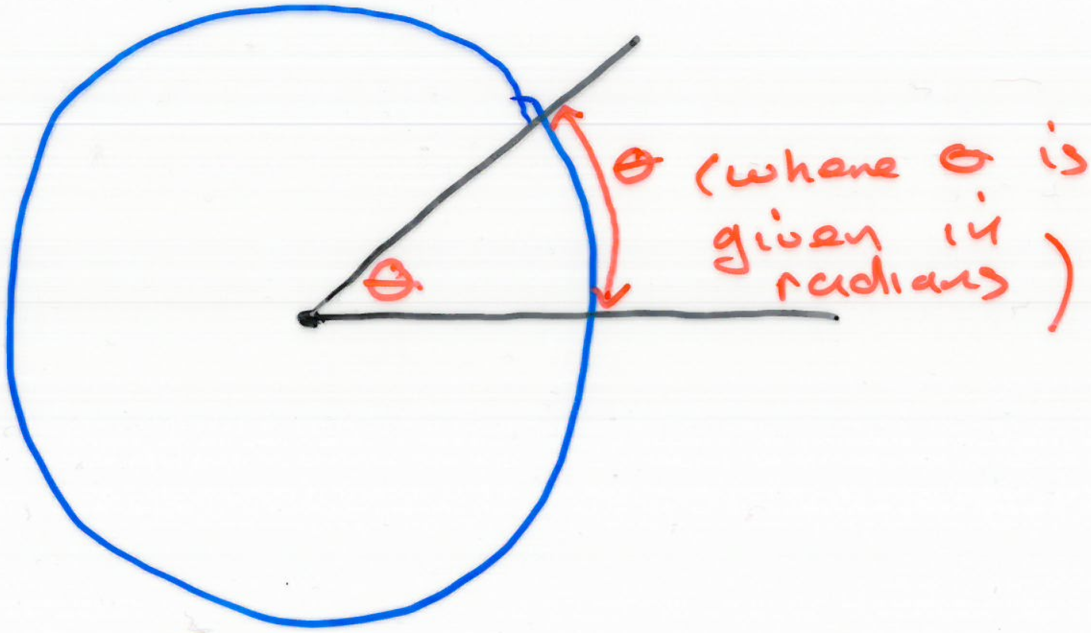
x-intercepts :

$x^2 - 1 = 0$

x-intercepts are the points $(-1, 0), (1, 0)$.

y-intercepts : $(0, -1)$.

What is a radian ?

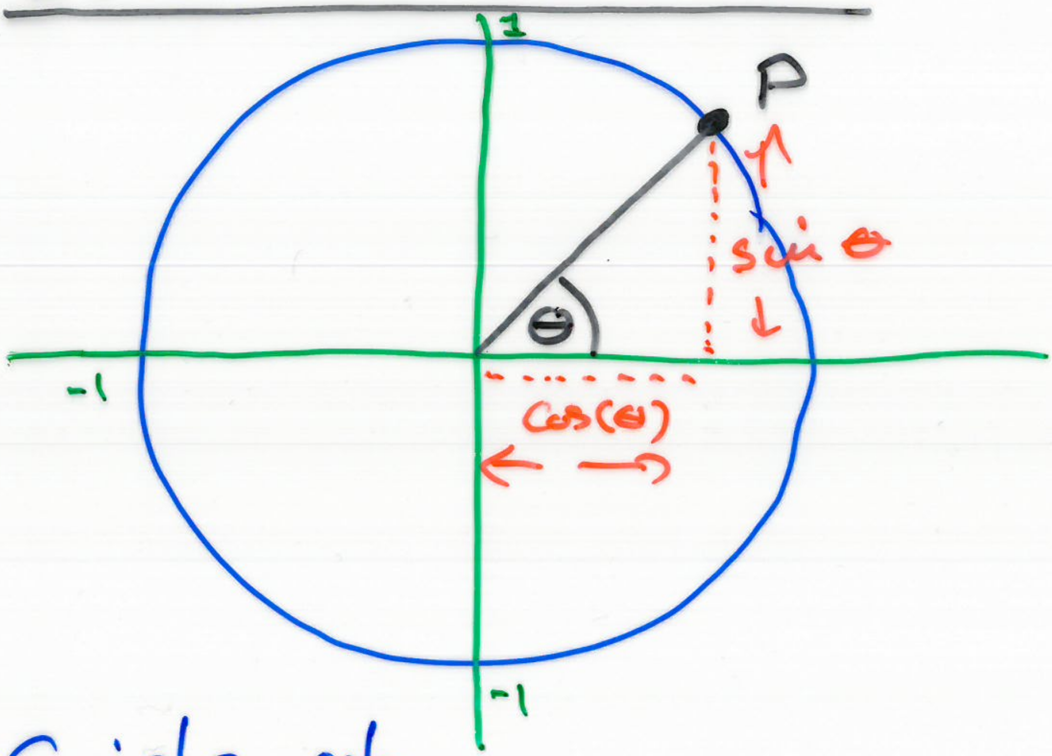


Circle of radius 1

$90^\circ = \frac{\pi}{2}$ radians

$180^\circ = \pi$ radians

Sine and Cosine



Circle of radius 1

P is the point $P = (\cos\theta, \sin\theta)$

Domain of $\sin(\theta) = \mathbb{R}$

codomain of $\sin(\theta) = \mathbb{R}$

Range of $\sin(\theta) = [-1, 1]$

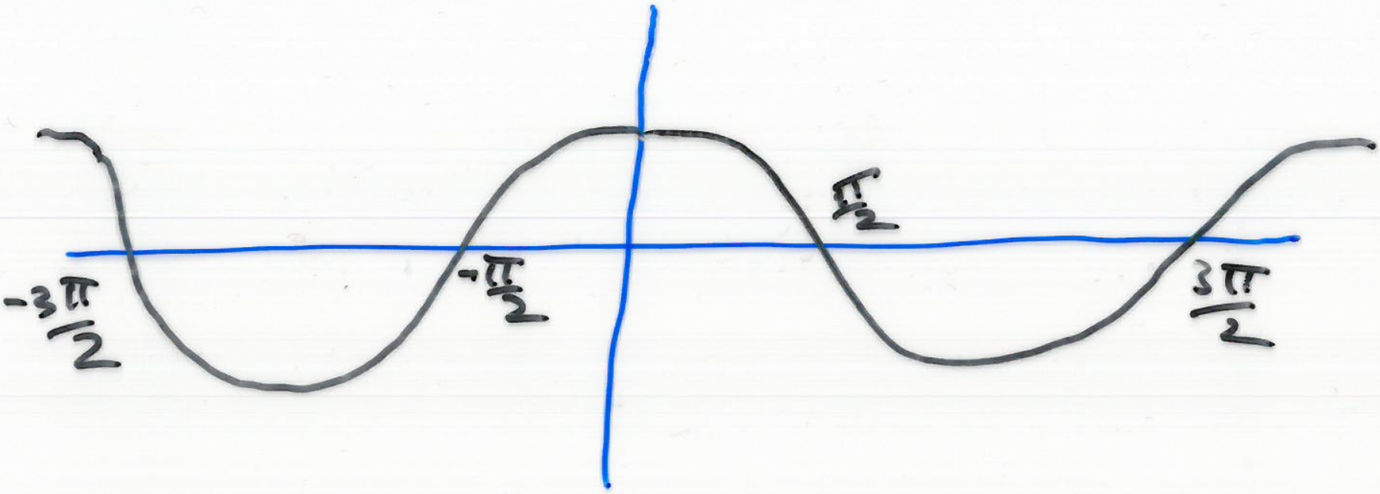
$\sin(-x) = -\sin(x)$

$\cos(-x) = \cos(x)$

$\sin(x + 2\pi) = \sin(x)$

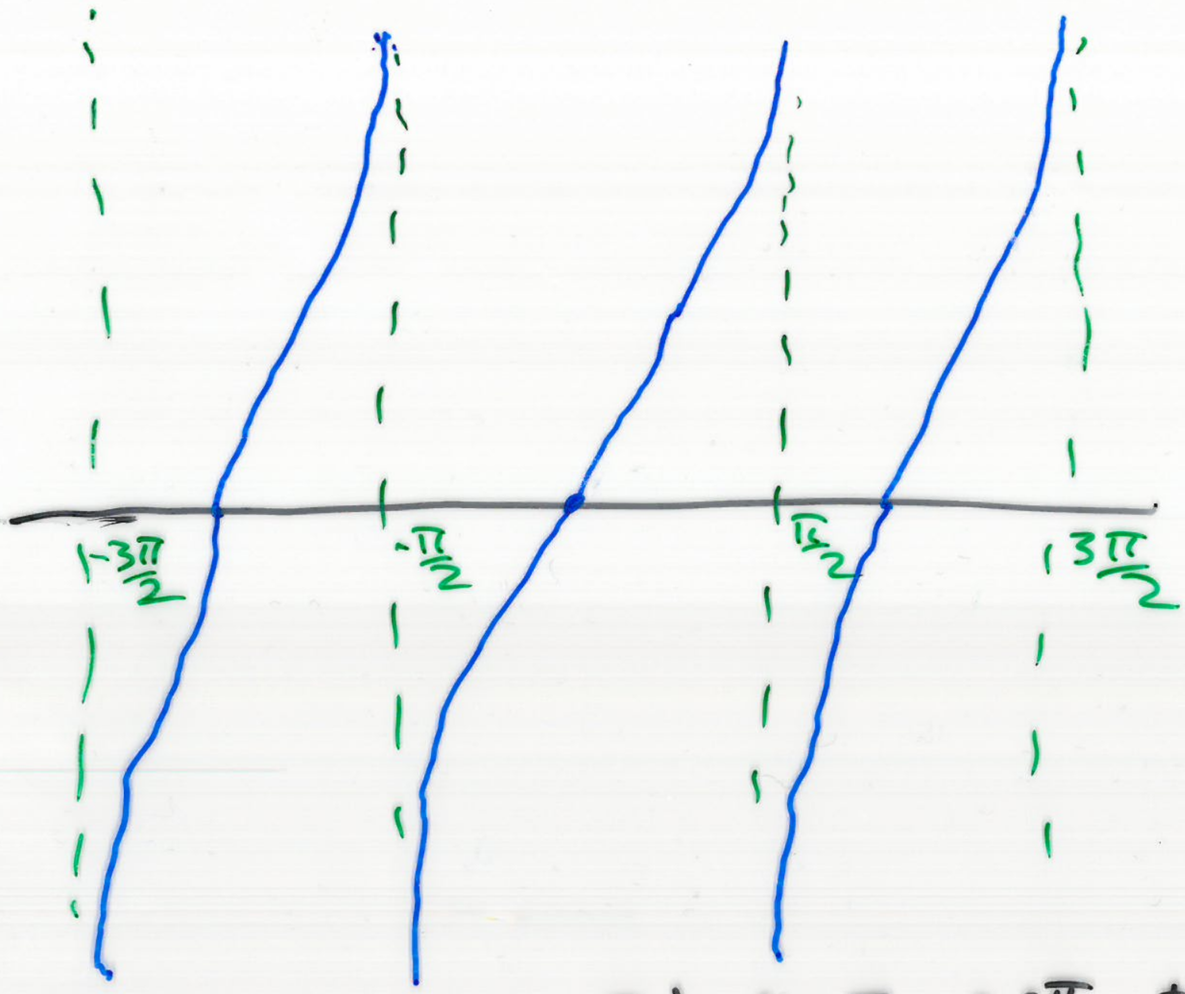
$\cos(x + 2\pi) = \cos(x)$

Graph of $\cos(x)$:



Definition:

$$\operatorname{Tan}(x) = \frac{\sin(x)}{\cos(x)}.$$



Domain of $\operatorname{Tan}(x) = \mathbb{R} \setminus \left\{ \pm\frac{\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{5\pi}{2}, \dots \right\}$