Q3 b)

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Don't use a red pen on my exam.

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Thus these are eigenvectors associated to eigenvalues $\lambda = -2$, $\lambda = 2$.

$$D = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$$

Eigenvector for $\lambda = -2$:

$$(A - \lambda I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is eigenvector for $\lambda = -2$.

<u>For $\lambda = 2$</u>

$$\begin{pmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. is an eigenvect. for $\lambda = 2$

$$E = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix}.$$

Intermediate Value Theorem

Suppose $f : [a, b] \to \mathbb{R}$ is continuous and $f(a) f(b) < 0$. Then $\exists$ $c \in [a, b]$ with

$f(c) = 0$.

Consider

$f(x) = x^3 - 4x - 1$

$x =$	-2	-1	0	1	2	3
$f(x)$	<0	>0	-1	-4	-1	>0

There is a solution in $[2, 3]$

" " " $[-1, 0]$

" " " $[-2, -1]$

No polynomial of degree n has more than 3 roots.

A critical point is where $f'(c) = 0$ or $f'(c)$ does not exist.

$$f'(x) = 4x^3 - 12x^2$$
$$= 4x^2(x-3)$$

critical points are when $x=0$, and $x=3$ i.e. the points $(0, 10)$ and $(12, f(3))$.

$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

$f'(x)$ $- - - - - - - - -$ 0 $- - - - - -$ 0 $+ + + + + + +$

$f''(x)$ $+ + + + + + + + + + +$ 0 $- - - -$ 0 $+ + + + + + + +$

$x=0$ and $x=2$ are points of inflection.

ii) f strictly decreases on $(-\infty, 0)$ and $(0, 3)$

f strictly increases on $(3, \infty)$.

iii) $x=0$ is neither a max nor min.

$x=3$ is a min.

iv) f is concave up on $(-\infty\ 0)$ and on $(2, \infty)$.

It is concave down on $(0, 2)$.

6 a)

$f(t) = t^{\frac{1}{2}}$ has anti-derivative

$$F(t) = \frac{2}{3} t^{\frac{3}{2}} .$$

$g(t) = t \sin(t^2)$ has antideriv.

$$G(t) = -\frac{1}{2} \cos(t^2)$$

$h(t) = \frac{1}{2t+1}$ has anti-derivative

$$H(t) = \frac{1}{2} \ln(2t+1)$$