Separable Differential Equations

A diff. eq^n is separable if it is of the form

$$f(y)\,\frac{dy}{dt} = f(t)$$

for functions $f(y)$, $g(t)$.

Example

$$y^2\,\frac{dy}{dt} = t^2$$

is separable.

Example

$$\frac{dy}{dt} = ky - ly^2$$

is separable, because it can be expressed as

$$\frac{1}{ky-ly^2}\,\frac{dy}{dt} = 1$$

A separable eq^n

$$\boxed{f(y)\,\frac{dy}{dt} = g(t)}$$

can be rewritten as

$$\boxed{\frac{d}{dt}F(y) = g(t)}$$

where $F(y)$ is any anti-derivative of $f(y)$.

Consequently

$$\boxed{\boxed{\underbrace{F(y)}_{=\int f(y)\,dy} = \int g(t)\,dt + C}}$$

solution to the diff. eq^n.

Example Solve

$$y^2 \frac{dy}{dt} = t^2$$

Sol^n

$$\int y^2 \, dy = \int t^2 \, dt + c$$

$$\frac{1}{3} y^3 = \frac{t^3}{3} + c$$

$$y^3 = t^3 + c$$

$$\boxed{y = (t^3 + c)^{\frac{1}{3}}}$$ solution.

Example Solve

$$e^y \frac{dy}{dt} - t - t^3 = 0$$

Sol^n

$$e^y \frac{dy}{dt} = t + t^3$$

$$\int e^y \, dy = \int t + t^3 \, dt$$

$$e^y = \frac{1}{2}t^2 + \frac{1}{4}t^4 + c$$

$$\ln(e^y) = \ln\left(\frac{t^2}{2} + \frac{t^4}{4} + c\right)$$

$$\boxed{y = \ln\left(\frac{t^2}{2} + \frac{t^4}{4} + c\right)}$$

Sol^n

Exercise Solve

$$\frac{dy}{dt} = 1 + y^2 \quad , \quad y(0) = 0 .$$

40% is the pass score.
you'll pass with $2\frac{1}{2}$ questions
done correctly.

Do not use a red pen

in exams.

Q1 2017

a) i)

$$\lim_{x \to 0} \frac{\sqrt{4+x} - 2}{x}$$

$$= \lim_{x \to 0} \frac{(\sqrt{4+x} - 2)}{(x)} \frac{(\sqrt{4+x} + 2)}{(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{\cancel{4} + \cancel{x} - \cancel{4}}{\cancel{x}(\sqrt{4+x} + 2)}$$

$$= \lim_{x \to 0} \frac{1}{\sqrt{4+x} + 2}$$

$$= \frac{1}{4}.$$

Q1 a) ii)

$$\lim_{x \to +\infty} \frac{1}{\sqrt{x^2+2x} - x}$$

$$= \lim_{x \to \infty} \frac{(\sqrt{x^2+2x} + x)}{(\sqrt{x^2+2x} - x)(\sqrt{x^2+2x} + x)}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{x^2+2x - x^2}$$

$$= \lim_{x \to \infty} \frac{\sqrt{x^2+2x} + x}{2x}$$

$$= \lim_{x \to \infty} \frac{\sqrt{1 + \frac{2}{x}} + 1}{2}$$

$$= 1.$$