Functions

A function $f : D \to C$ consists of

1) a set D called the domain

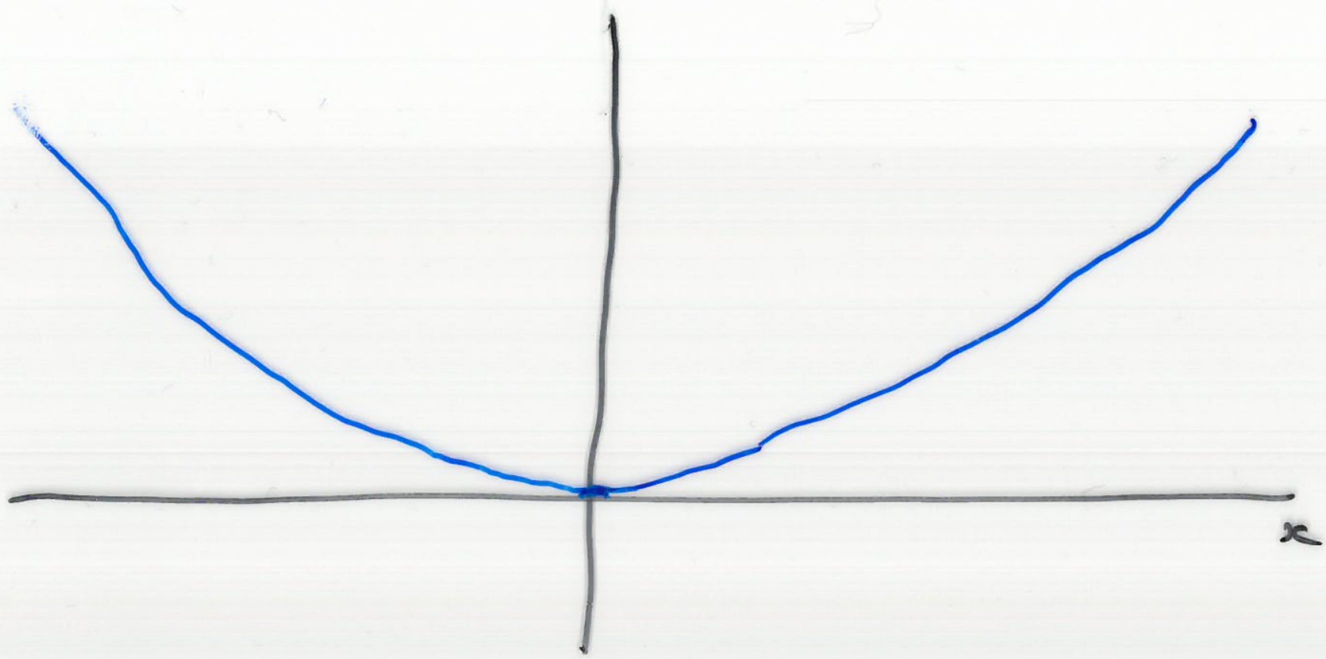
2) a set C called the codomain

3) a rule that assigns precisely one element $f(x) \in C$ to each $x \in D$.

Example $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = x^2$.

$D = \mathbb{R}$, $C = \mathbb{R}$

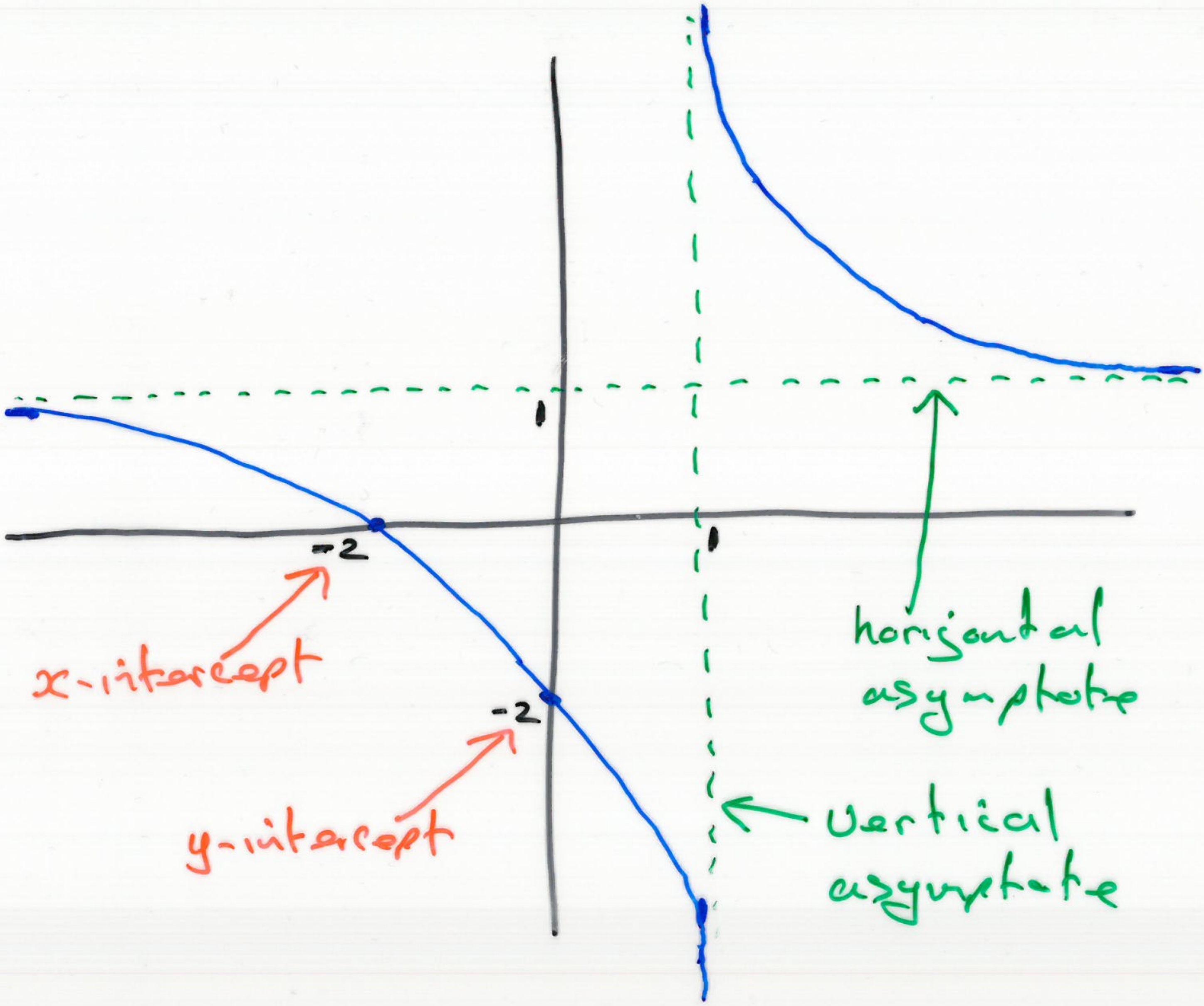
its graph is :



Example $g : \mathbb{R} \setminus \{1\} \longrightarrow \mathbb{R}$

with $g(x) = \dfrac{x+2}{x-1}$.

$D = \mathbb{R} \setminus \{1\}$, $C = \mathbb{R}$



Convention

Often we use a formula such as

$$h(x) = \frac{x+2}{x-3}$$

to describe a function, without explicitly stating a domain and codomain.

In this situation we always assume that the domain is the largest possible subset of $\mathbb{R}$ for which the formula makes sense.

We take the codomain to be $\mathbb{R}$.

Examples

1) $h(x) = \dfrac{x+2}{x-3}$

$D = \mathbb{R} \setminus \{3\}$

$C = \mathbb{R}$

2) $q(x) = \sqrt{x}$ positive square root of x.

$D = \{x \in \mathbb{R} : x \geq 0\}$

$= [0, \infty)$ $\qquad C = \mathbb{R}$.

3) $p(x) = \sqrt{x^2 - 1}$

$D = \{x \in \mathbb{R} : x \leq -1\} \cup \{x \in \mathbb{R} : x \geq 1\}$

$C = \mathbb{R}$

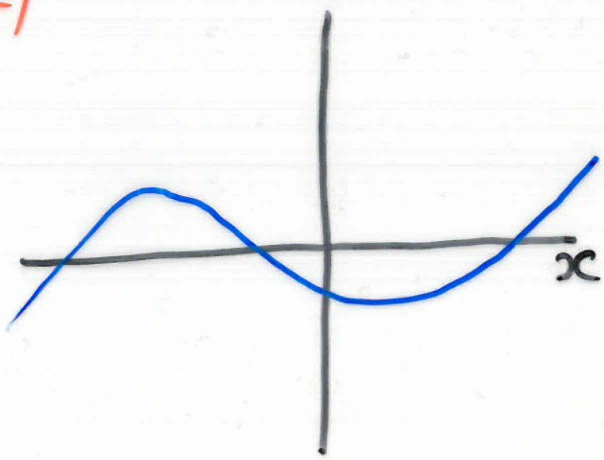
4) $r(x) = \sqrt{\frac{1}{x}}$

$D = \{x \in \mathbb{R} : x > 0\}$

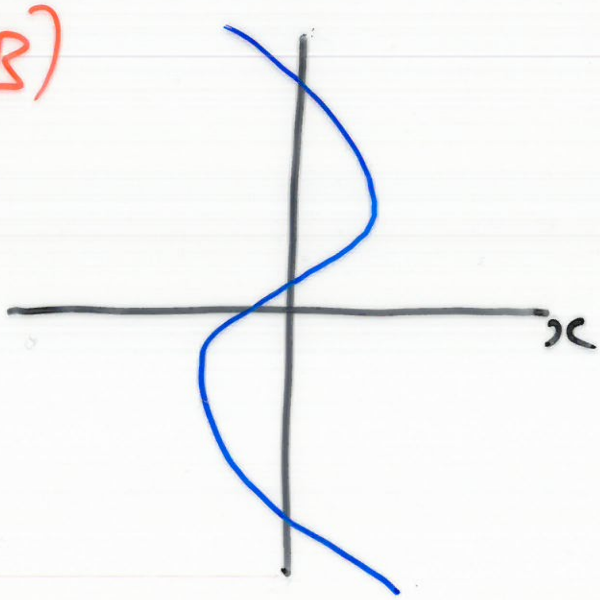
$= (0, \infty)$

Question which of the following are graphs of functions of x ?

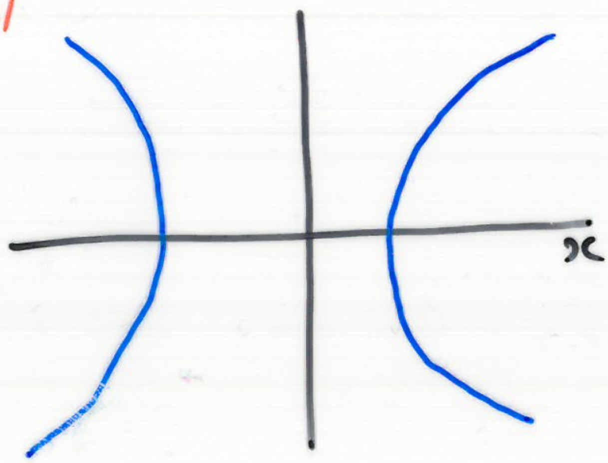
(A)



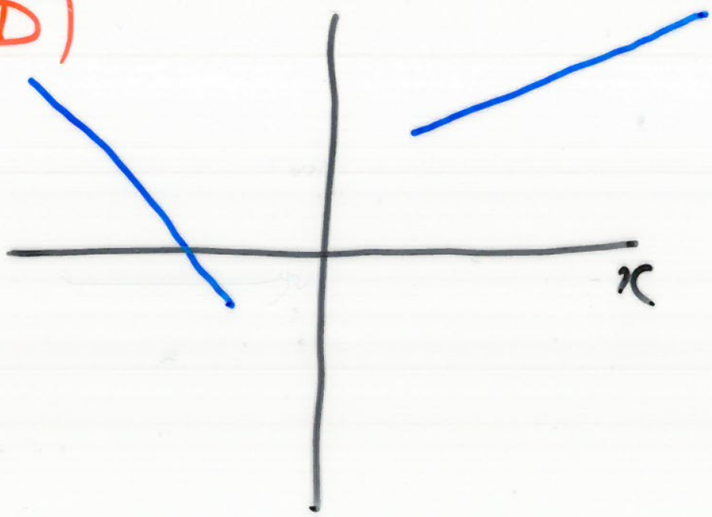
(B)



(C)



(D)



B is not a function

C is not a function

A is a function

D is a function.

The range of a function $f: D \to C$ is the set

$$\mathrm{Range}(f) = \{ y \in C : y = f(x) \text{ with } x \in D \}$$

Continuing with Examples 1-4 above:

$$\mathrm{Range}(h) = \mathbb{R} \setminus \{1\}$$

$$\mathrm{Range}(q) = \{ x \in \mathbb{R} : x \geq 0 \} = [0, \infty)$$

$$\mathrm{Range}(p) = \{ x \in \mathbb{R} : x \geq 0 \} = [0, \infty)$$

$$\mathrm{Range}(r) = \{ x \in \mathbb{R} : x > 0 \} = (0, \infty)$$