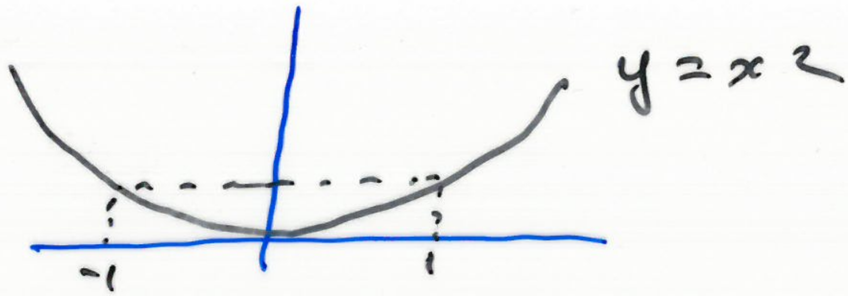
A function $f : D \to \mathbb{R}$ is said to be <u>injective</u> if

$f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in D$.

<u>Example</u> a) $f(x) = x^2$, with $D = \mathbb{R}$.

This is <u>not</u> injective because, for instance, $f(1) = f(-1)$.



<u>Example</u> b) $f(x) = x^3 - 2$.

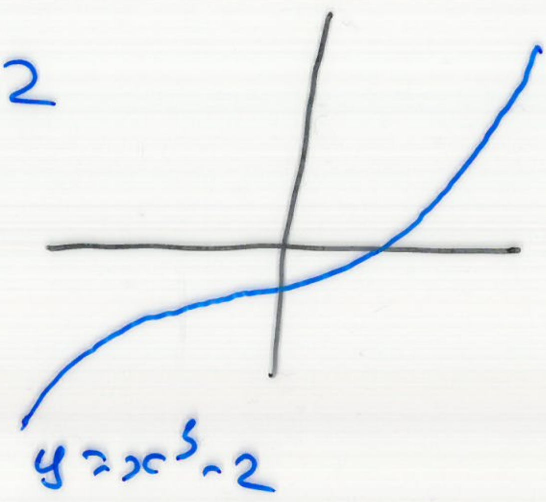
If $f(x_1) = f(x_2)$

then $x_1^3 - 2 = x_2^3 - 2$

then $x_1^3 = x_2^3$

and so $x_1 = x_2$.

So $f(x)$ <u>is</u> injective.



Suppose that $f : D \to \mathbb{R}$ is an injective function. Then the **inverse function** f^{-1} is defined by the rule

$$f^{-1}(y) = x \quad \text{precisely when} \quad y = f(x).$$

The domain of f^{-1} is equal to the range of f.

Example Find the inverse of $f(x) = x^3 - 2$, with domain $D = \mathbb{R}$.

Sol^n

$$y = x^3 - 2$$

$$y + 2 = x^3$$

$$\sqrt[3]{y+2} = x.$$

So $f^{-1}(x) = \sqrt[3]{x+2}$ with domain $D = \mathbb{R}$.

Observe

$$\boxed{f^{-1}(f(x)) = x} \qquad (*)$$

<u>Proposition</u> (Derivative of an inverse function)

Suppose

$$f'(f^{-1}(x)) \neq 0$$

for any x.

Then

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

<u>Proof</u> $f^{-1}(f(x)) = x$

Differentiate both sides (using the chain rule on the left-hand side)

$$(f^{-1})'(f(x))\, f'(x) = 1$$

Now write $y = f(x)$ or $x = f^{-1}(y)$, to get

$$(f^{-1})'(y) \cdot f'(f^{-1}(y)) = 1$$

and

$$(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))} .$$

<u>Example</u> If $f(x) = x^3 - 2$

then

$$f^{-1}(x) = \sqrt[3]{x+2}$$

from the proposition

$$(f^{-1})'(x) = \frac{1}{3(\sqrt[3]{x+2})^2}.$$

Exercise: check this by differentiating $f(x) = x^3 - 2$ directly.

Back to logarithms

from the definition (yesterday)

of the function

$$y = \ln(x)$$

we see that $\ln(x)$ is injective.

so $y = \ln(x)$ has an inverse function, which we denote

by $\exp(x)$ or e^x.

we can define

$$\log_4(\sqrt{2}) = \frac{\ln(\sqrt{2})}{\ln(4)} .$$

Recall

$$\arcsin(y) = x$$

means

$$\sin(x) = y.$$

we also write $\sin^{-1}(x)$ for $\arcsin(x)$.

Example Find $\frac{dy}{dx}$ where

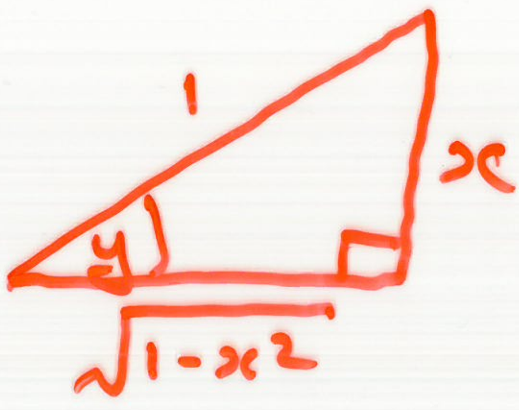
$$y = \sin^{-1}(x).$$

Sol^n $x = \sin(y)$

Differentiate both sides w.r.t. x

$$1 = \cos(y) \frac{dy}{dx}$$

so $\frac{dy}{dx} = \frac{1}{\cos(y)}$



$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

NAME	MA180	MA185	MA 190
CIAN DILLANE			✓
Luke Finn	✓		
James Spillane		✓	