

Logarithms

A logarithm is a function

$$\ln : (0, \infty) \longrightarrow \mathbb{R}$$

satisfying

$$\bullet \ln(xy) = \ln(x) + \ln(y)$$

$$\bullet \ln(x^n) = n \ln(x)$$

there must be an associated function

$$\exp : \mathbb{R} \longrightarrow (0, \infty)$$

satisfying

$$\ln(\exp(x)) = x$$

sometimes \exp is called an

anti logarithm.

when he was in Primary School:
to multiply

1.23456

by

7.89123

he'd just calculate

$$a = \ln(1.23456)$$

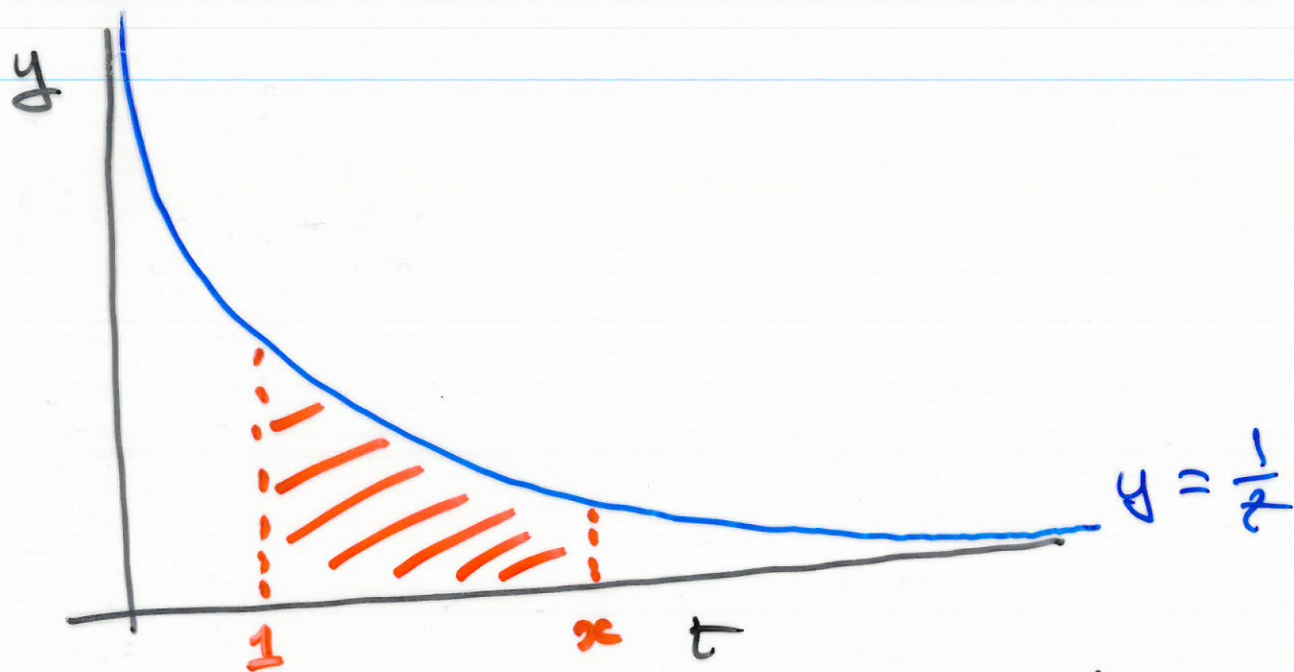
$$b = \ln(7.89123)$$

and then

$$\text{Exp}(a + b) = 1.23456 + 7.89123$$

Today's aim: he wants to
rigorously construct/define
a logarithm function.

Defn For $x > 0$ let A
be the area



between the curve $y = \frac{1}{t}$ and
the t -axis from $t=1$ to $t=x$,

we define

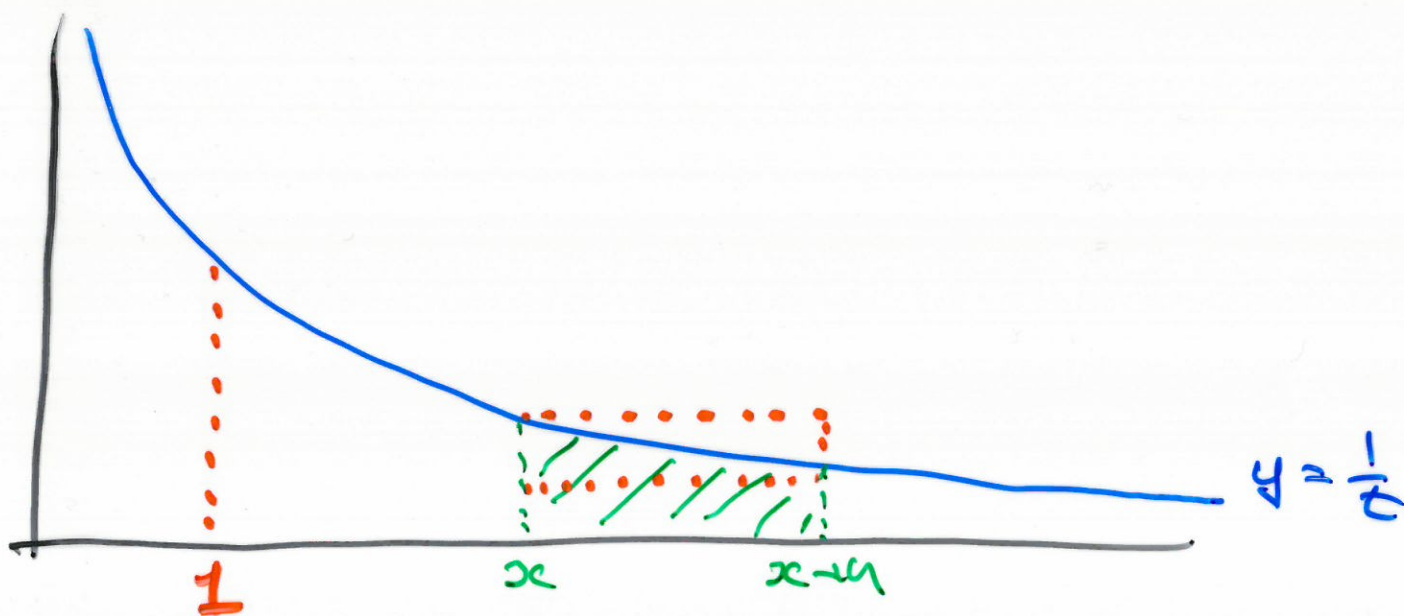
$$\ln(x) = \begin{cases} A & \text{if } x \geq 1 \\ -A & \text{if } 0 < x < 1 \end{cases}$$

we call $\ln(x)$ the natural
logarithm.

Theorem If $x > 0$ then

$$\frac{d}{dx} \ln(x) = \frac{1}{x} .$$

Proof



$$\frac{h}{x+h} < \text{Shaded green area} < \frac{h}{x}$$

Dividing throughout by h yields:

$$\frac{1}{x+h} < \frac{\ln(x+h) - \ln(x)}{h} < \frac{1}{x}$$

$$\text{So } \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln(x)}{h} = \frac{1}{x} \quad (\text{Sandwich Lemma})$$

Then

$$\frac{d}{dx} \ln(x) = \frac{1}{x}.$$

QED

Consequence 1 For $x, y > 0$

we have

$$\ln(xy) = \ln(x) + \ln(y).$$

Proof

$$\frac{d}{dx} (\ln(xy) - \ln(x))$$

$$= \frac{y}{xy} - \frac{1}{x}$$

$$= 0.$$

Thus

$$\ln(xy) - \ln(x) = c \quad (*)$$

c is a constant
not depending on
 x .

Put for instance $x = 1$ in (*),
we see

$$\ln(y) - \ln(1) = C$$

and $\ln(y) = C.$

Then, from (*),

$$\ln(xy) - \ln(x) = \ln(y)$$

or

$$\ln(xy) = \ln(x) + \ln(y).$$

QED

We could also easily show
that

$$\ln(x^n) = n \ln(x).$$

Past exam question

Find the derivative y' of

$$y = \frac{(x+1)(x+2)(x+3)}{(x+4)}$$

Soln

$$\ln(y) = \ln(x+1) + \ln(x+2) + \ln(x+3) - \ln(x+4)$$

Differentiate both sides with respect to x .

$$\frac{1}{y} y' = \frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} - \frac{1}{x+4}$$

Thus

$$y' = \frac{(x+1)(x+2)(x+3)}{(x+4)} \left\{ \frac{1}{x+1} + \frac{1}{x+2} + \frac{1}{x+3} - \frac{1}{x+4} \right\}$$

NAME

MA18

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Evelyn Gutebel - 18469456

X

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X

Lydia Costello - 18320966

X

Anna Golden - 18367341

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Emma Meoney - 18419734

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Kate Campbell - 18325516

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Ciaran Rafferty - 18474872

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Conor Mc Kiernan - 18495206

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Fionnuala Jorkan - 18389341

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Michael McGloin 18357636

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Patrick Ryder - 18303583

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CATHAL BOYCE 18376036

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Paul O'Loughlin 18424102

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Marie Murphy 17101928

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Alex Moserell 1832086

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Helly Rody 18455416

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Ruth Manning 18314443

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| David O'Dea | 18468766 |
| Dionuid Donnellan | 18465640 |
| Eoin Mulvihill | 18389281 |
| Ross Hearty | 18317711 |
| Lise Wall | 18311086 |
| Lauren O'Malley | 18309423 |
| Rachel Kelly | 18354566 |
| Sam Hudson | 18467574 |
| Saire Ni Mhuilleán | 18317401 |
| Ronan Finegan | 18775361 |
| Lloyd Boulger | 18762351 |
| Liam Doherty | 18429174 |
| Jack Sullivan | 18505716 |
| Liam English | 18358123 |

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| Yeo Zhu Hao | 18280074 | | X | |
| Rajath Raja | 18280094 | | X | |
| Eoin McArdle | 18413686 | | X | |
| Cathal O'Callaghan | 18365213 | | X | |
| Bory Sweeney | 18342673 | | X | |
| Oisín M'Glin | 18357641 | X | | |
| Karl Gordon | 18321516 | | X | |
| Ryan Kidd | 18284966 | X | | |
| Nicolas Amaya Aguilar | 18394023 | X | | |
| Robert Deery | 18369573 | X | | |
| Enda Tynan Daly | 18357483 | X | | |
| Eoghan Brohey | 18417212 | X | | |
| Josh Stoney | 12309021 | X | | |
| Walter Krawczyk | 17103718 | X | | |
| Micheál Leddy | 18448004 | X | | |
| Claire Moloney | 18308936 | X | | |
| Enloe McDermott | 18336233 | X | | |
| Liam Lupton-Smith | 18341696 | X | | |
| Jan Konrad Krzyński | 18376701 | | | X |
| Labhaoise Barrett | 18321786 | | | X |
| Eoin Callinan | 18497564 | X | | |
| Conor McGovern | 18343763 | | X | |
| Lawren O'Donnell | 18302633 | | X | |
| Ciara Galvin | 18344346 | | X | |
| Rory O'Halloran | 18385776 | | | X |
| Sarah Gavin | 18447766 | X | | X |
| Orlaith Ni Chonaire | 18384931 | | | X |
| Mark Ryder | 18390276 | X | | |
| Anton Sohn | 18464724 | X | | |
| Tiarnan Boyle | 18433722 | X | | |

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| Name | MA180 | MA190 | MA185 |
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| Shane Maher | ✓ | | |
| Phooi Tyng See | | | ✓ |
| Meg Regan | | | ✓ |
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| Adem O'Reilly | 18429872 | ✓ | | |
| Ermet Connolly | 18340726 | ✓ | | |
| Bailey Kane | 18458664 | ✓ | | |
| Conor Fogarty | 18358401 | | ✓ | |
| Ellie O'Bole | 18323523 | ✓ | | |
| Fiachra Healy | 18365186 | | ✓ | |
| Paul Costello | 18376376 | ✓ | | |
| Stephen Durlan | 18303483 | ✓ | | |
| Ruth Kelly | 18302426 | ✓ | | |
| Orla O'Neill | 18314251 | ✓ | | |
| Sack Flood | 18337526 | ✓ | | |
| Gada O'Connor | 18428292 | ✓ | | |
| Dara Carr | 18483836 | ✓ | | |
| Jenna Hopkins | 18436874 | ✓ | | |
| Karen Gullen | 18353391 | ✓ | | |
| ORAN HESTER | 18476694 | ✓ | | |
| MIHALO SRECKOVIC | 18738401 | ✓ | | |
| Katie Feeney | 18323456 | MM | | ✓ |
| Chloe O'Reilly | 18339573 | | | ✓ |
| Casey Cowan | 17360043 | | | ✓ |
| Sinead St. John | 18400884 | | | ✓ |
| Darragh McCrawford | 18399636 | ✓ | | |
| Ciara Hamilton | 18353123 | | | ✓ |
| Sharon Donohoe | 18416682 | | | ✓ |
| Mary O'Grady | 18329466 | ✓ | | |
| Cian Mac Coille | 18464906 | ✓ | | |
| Catherine Fraser | 18420754 | ✓ | | |
| Sinead Bannon | 18759869 | ✓ | | |
| Sean Tynan | 18341103 | ✓ | | |
| Anna Mc Nena | 18359021 | ✓ | | |

| Name | | MA180 | MA190 | MA185 |
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| Siobhan Beatty | 18331526 | ✓ | | |
| Megan Mc Glinchen | 18902362 | ✓ | | |
| Sophie Plunkett | 18491924 | ✓ | | |
| David Flanagan | 18452262 | ✓ | | |
| Julia Ingles | 18719789 | ✓ | | |
| Caomhe Reidy | 18319833 | ✓ | | |
| Megan Reilly | 18326546 | ✓ | | |
| Oisín O'Leary | 18457204 | ✓ | | |
| Rory Delaney | 18446072 | ✓ | | |
| Jack Done | 18392921 | | ✓ | |
| Patrick Scully | 18740211 | | ✓ | |
| Seamus O'Leary | 18301096 | | ✓ | |
| Shain Sutton | 18331093 | | ✓ | |
| Cian Gibbons | 18325633 | | ✓ | |
| Stephen MacDonnadh | 18314581 | | ✓ | |
| David Noone | 18727451 | | ✓ | |
| Nicholas Correntino | 18331203 | ✓ | | |
| Marie McDonagh | 18398886 | | | ✓ |
| Eleanor Fitzpatrick | 18329551 | | | ✓ |
| Darren Browne | 18385273 | ✓ | | |
| Peadar O'Donnell | 18374076 | ✓ | | |
| Michael Dowling | 184063854 | ✓ | | |
| Evan Fahy | 18341616 | | ✓ | |
| Evan O'Riordan | 18425352 | | ✓ | |