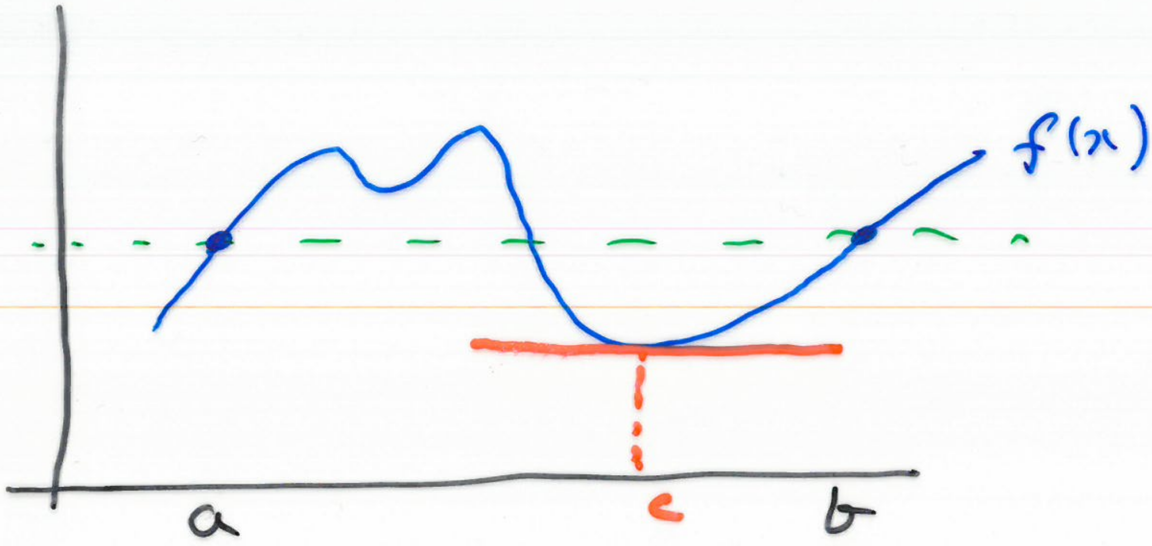


<u>Theorem</u> (Rolle)

if $f(x)$ is continuous at all points in $[a, b]$, and if $f(x)$ is differentiable at all points in (a, b), and if $f(a) = f(b)$ then there exists at least one point $c \in (a, b)$ such that

$$f'(c) = 0.$$

__Exercise__ Prove that there is exactly one solution to

$$x^3 + x + 1 = 0. \qquad (*)$$

__Sol^n__

Let $f(x) = x^3 + x + 1$

$f(-1) < 0$

$f(0) > 0$

So the IVT says there is ~~at least~~ once $c \in [-1, 0]$ such that $f(c) = 0$.

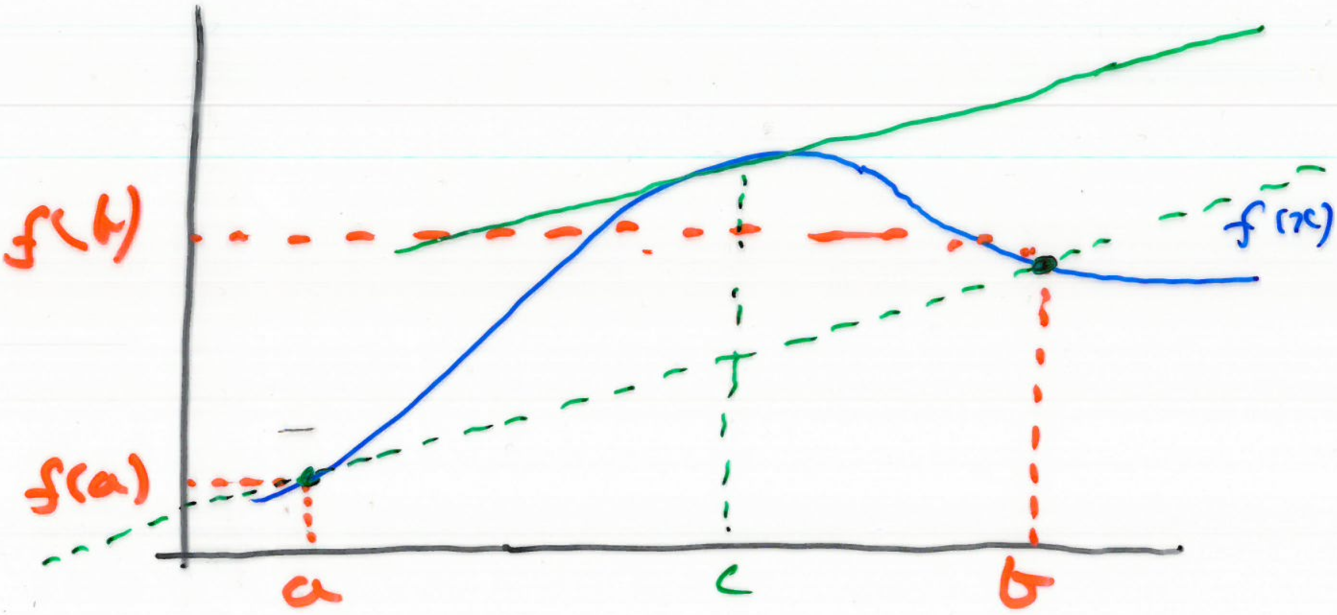
Hence $(*)$ has at least one solution.

Consider $f'(x) = 3x^2 + 1$.

Note that $f'(x) > 0$ for all x.

so by Rolle's Theorem we can't have $f(d) = 0$ for any other $d \neq c$.

The Mean Value Theorem



Theorem Suppose that

$f : [a, b] \longrightarrow \mathbb{R}$ is continuous on $[a, b]$ and differentiable on (a, b). Then there exists at least one point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Logarithms & Exponents

$4^2 = 16$	$\log_4 16 = 2$
$4^3 = 64$	$\log_4 64 = 3$
$4^{\frac{1}{2}} = \sqrt{4} = 2$	$\log_4 2 = \frac{1}{2}$
$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$	$\log_4 \frac{1}{16} = -2$
$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8$	$\log_4 8 = \frac{3}{2}$
$4^{\frac{5}{2}} = \left(4^{\frac{1}{2}}\right)^5 = 2^5 = 32$	$\log_4 32 = \frac{5}{2}$

We can make sense of 4^n when n is an integer $n = 0, \pm 1, \pm 2, \pm 3, \cdots$ and when n is a fraction

$$4^{\frac{p}{q}} = \left(\sqrt[q]{4}\right)^p .$$

Terminology: Fractions of the form $\frac{p}{q}$ with p, q integers are called rational numbers.

what does $3^{\sqrt{2}}$ mean ? we'd like to treat exponents (and logarithms) carefully so that we have a meaning for $3^{\sqrt{2}}$.

Is $(\sqrt{2})^{\sqrt{2}}$ rational or not ?

I have no idea !

Properties of logarithms

$$\boxed{y = a^x \iff \log_a y = x}$$

$$\boxed{(a^m)^n = a^{mn}} \quad 1$$

$$\boxed{a^m a^n = a^{m+n}} \quad 2$$

Suppose

$$u = a^m, \quad v = a^n$$

$$\begin{aligned}
\log_a(uv) &= \log_a(a^m a^n) \\
&\overset{2}{=} \log_a(a^{m+n}) \\
&= m + n \\
&= \log_a u + \log_a v
\end{aligned}$$

"Thus"

$$\boxed{\log_a(uv) = \log_a(u) + \log_a(v)}$$ 2'

Now let $u = a^m$.

Then

$$\log_a u^n = \log_a((a^m)^n)$$

$$\underset{1}{=} \log_a(a^{mn})$$

$$= mn$$

$$= n\log_a(u)$$

"Thus"

$$\boxed{\log_a(u^n) = n\log_a(u)}$$ 1'