Q4b)

The function $g(x)$ is clearly continuous at all $x \neq 1$.

For continuity at $x=1$ we need

$$x^3 + kx^2 + x = lx + k \quad \text{with } x=1.$$

i.e. we need

$$2 + k = l + k$$

So $\boxed{l = 2}$

For $g(x)$ to be differentiable at $x=1$ we need

$$\frac{d}{dx}(x^3 + kx^2 + x) = \frac{d}{dx}(lx + k) \quad \text{with } x=1.$$

i.e. $3x^2 + 2kx + 1 = l$ with $x = 1$.

i.e. $4 + 2k = l$

Do not use a red pen in my exam

So $k = -1$.

Q1 (a)

i) $37 = 2.14 + 9$

$14 = 9 + 5$

$9 = 5 + 4$

$5 = 4 + (1) = \gcd(37, 14)$

$1 = 5 - 4$

$= 5 - (9-5) = 2.5 - 9$

$= 2(14-9) - 9 = 2.14 - 3.9$

$= 2.14 - 3(37 - 2.14)$

$\equiv 8.14 \mod 37$

$14^{-1} \equiv 8 \mod 37$

Q1(a)

ii) $f_E : x \longmapsto 14x + 20 \mod 37$

$f_D : x \longmapsto 14^{-1}(x-20)$

$= 8(x-20) \mod 37$

$= 8x - 12$

iii) $H = 17$

$V = 31$

$f_D(H) = f_D(17) = 8.17 - 12$

$= -12 - 12$

$= -24$

$= 13$

$= D$

$f_D(V) = f_D(31) = 8.31 - 12$

$= -11 - 12$ Plaintext is

$= -23$ DEED

$= 14$

$= E$

Q1(b) (i)

$$360 = 2.180$$
$$= 2^2 . 90$$
$$= 2^3 . 45$$
$$= 2^3 . 3 . 15$$
$$= 2^3 . 3^2 . 5$$

Q1b (ii)

$$\phi(360) = \phi(2^3 . 3^2 . 5)$$
$$= \phi(2^3)\,\phi(3^2)\,\phi(5)$$
$$= (2^3 - 2^2)(3^2 - 3)(5-1)$$
$$= 4 . 6 . 4 = 96 .$$

Q1b(iii) Remember $a^{\phi(m)} \equiv 1 \mod m$ assuming $\gcd(a, m) =$

$$11^{98} = 11^{96}\, 11^2$$
$$\equiv 11^2 \mod 360$$
$$= 121 .$$