$$\begin{rcases} \boxed{2}x + 3y + 2z = 100 \\ x + y + 4z = 70 \\ 20x + 10y + 10z = 500 \end{rcases} \text{system of linear equations}$$

The system is equivalent to the following system.

$$\begin{bmatrix} R_2 \longmapsto R_2 - \frac{1}{2}R_1 \\ R_3 \longmapsto R_3 - 10R_1 \end{bmatrix}$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ \boxed{-\tfrac{1}{2}}y + 3z &= 20 \\ -20y - 10z &= -500 \end{aligned}$$

This second system is equivalent to

$$\begin{bmatrix} R_3 \longmapsto R_3 - 40R_2 \end{bmatrix}$$

$$\begin{aligned} 2x + 3y + 2z &= 100 \\ -\tfrac{1}{2}y + 3z &= 20 \\ -130z &= -1300 \end{aligned}$$

Back substitution:

$z = 10$

$y = 20$

$x = 10$

Notation

2 is the pivot in the first stage

$-\frac{1}{2}$ is the pivot in the second stage.

The above procedure for solving a system of linear equations is called gaussian elimination.

Q) Does this procedure always work?

A) No, it doesn't work if a pivot of some stage is zero.

Topic 3

Determinants, eigenvalues & eigenvectors.

(Mainly 2x2 matrices)

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Definition The adjoint matrix of A is

$$\mathrm{adj}(A) = \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Observe:

$$A.\,\mathrm{adj}(A) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix} = (ad-bc)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Definition The determinant of A is the number

$$\det(A) = ad - bc.$$

Note:

$$A \cdot \operatorname{adj}(A) = \det(A)\, I$$

or

$$A \cdot \left(\frac{1}{\det(A)} \operatorname{adj}(A) \right) = I$$

"thus"

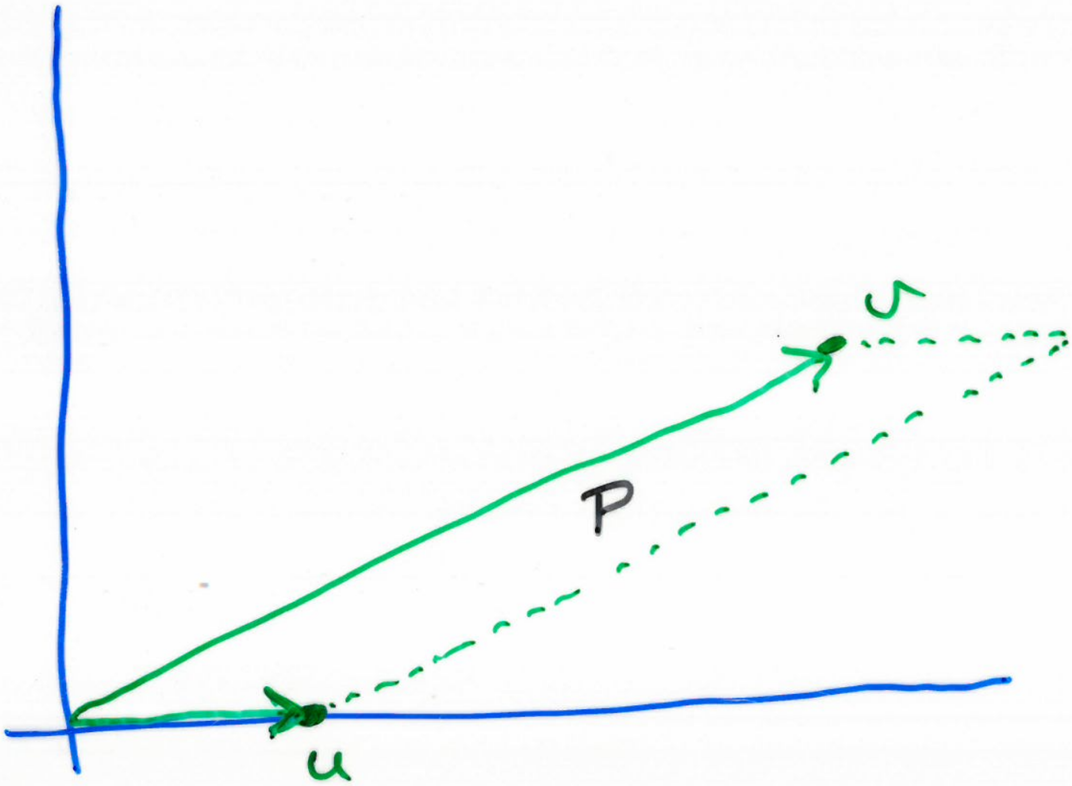
Proposition if $\det(A) \neq 0$ then

$$A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)$$

for any 2×2 matrix A.

Consider two "random" vectors

$$u = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad v = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$$



So u and v determine a parallelogram P,

Area of P = base $\times$ $\perp^r$ height
$= 2 \times 3$
$= 6$

Consider

$$A = \begin{pmatrix} 2 & 6 \\ 0 & 3 \end{pmatrix}$$

The vectors u, v can be thought of as the columns of the matrix A.

$$\det(A) = 2.3 - 0.6 = 6.$$

<u>Theorem</u> The determinant of a 2×2 matrix is equal to the $\pm$ area of the parallelogram determined by its two columns.

Notation

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det(A) = ad - bc$$

or $|A| = ad - bc$

Example

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} -1 & -2 \\ 3 & -4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 7 & -16 \\ 11 & -18 \end{pmatrix}$$

$$|AB| = 7(-18) - (11)(-16) = 50$$

$$|A| = 2 \cdot 4 - 1.3 = 5$$

$$|B| = (-1)(-4) - (3)(-2) = 10$$

Observe:

$$|AB| = |A|\,|B|$$

$$50 \qquad 5 \cdot 10$$