MA180, MA185, MA190

Algebra & Calculus

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Algebra

Topics	Context
• Elementary number Theory	Internet communications
• Matrix Theory	Geometry & Internet communications
• Eigenvectors & Eigenvalues	Breeding rabbits

Elementary Number Theory

$5 + 15 = 20$ In School

$7 + 10 = 5$ on a clock

$7 + 10 \equiv 5 \mod 12$

Today is Monday. In 73 days time it will be Thursday.

$1 + 73 \equiv 4 \mod 7$

More examples

$10 \times 5 \equiv 2 \mod 12$

$$7 + 5 \equiv 3 \mod 9$$

$$7 \times 8 \equiv 2 \mod 9$$

$$2 - 5 \equiv 5 \mod 8$$

What is $\frac{1}{3}$?

What is $1 \div 3$?

$\frac{1}{3}$ is that number with the property

$$\left(\frac{1}{3}\right) \times 3 = 1$$

Alternative notation

$$3^{-1} = \frac{1}{3}$$

we call 3^{-1} the inverse of 3.

Back to clocks

What is $7^{-1} \mod 10$?

$$7^{-1} \equiv 3 \mod 10$$

because $7 \times 3 \equiv 1 \mod 10$

Applications

Any book is identified by an ISBN. On older books this is a string of 10 digits.

0 5 5 2 1 2 4 7 5 3

Terry Pratchett

The final digit is a safety check digit. It is chosen so that

$(1\times0)+(2\times5)+(3\times5)+(4\times2)+(5\times1)$

$+(6\times2)+(7\times4)+(8\times7)+$

$(9\times5)+(10\times3)$

$\equiv 0-1+4-3+5+1+6+1+1+8 \mod 11$

$\equiv 0 \mod 11$

Second Example

0 1 4 y 2 2 5 8 9 7

$1.0+2.1+3.4+4.y+5.2+6.2+7.5+8.8+9.9+10.7$

$\equiv 2+1+4y-1+1+2-2+4+4 \mod 11$

$\equiv 0+4y \mod 11$

Hence $y=0$.