Semester I Examinations 2011/2012

Exam Code(s)	2BMS1, 2BPA1, 2BPM1, 2BPP1, 2BS1, 3BS9/ 2FM1/ 2BCT1
Exam(s)	Second Arts, Science and Engineering
Module(s)	Mathematical Methods I / Methods of Mathematical Physics I
Module Code(s)	MP231/MP291/MP209
Paper No	1
Repeat Paper	
External Examiner(s)	Prof. G. Saccomandi
Internal Examiner(s)	Prof. M. Destrade
	*Dr. Michael Mc Gettrick
<u>**Instructions:**</u>	**Answer any three (3) questions**
	All questions are marked equally
Duration	2 Hours
No. of Pages	3
Discipline(s)	Applied Mathematics
Course Co-ordinators(s)	

<u>**Requirements:**</u>

MCQ	
Handout	
Statistical Tables/ Log Tables	Yes
Cambridge Tables	
Graph paper	
Log Graph Paper	
Other Materials	

1. (a) Use the chain rule to calculate $f_t = \mathrm{d}f/\mathrm{d}t$ in each of the following cases:

   i. $f(x, y, z) = x^2 + y^2 - z$ while $x = t^3 - 1,\ y = 2t,\ z = 1/(t-1)$.

   ii. $f(x, y, z) = xyz$ while $x = e^{-t}\sin t,\ y = e^{-t}\cos t,\ z = t$.

(20 marks)

(b) Let $f(x, y) = Ax^4 + Bx^2y^2 + Cy^4$ where $A,\ B$ and C are arbitrary constants. Determine the value of (the number) p for which

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = pf$$

(13 marks)

2. (a) Find all stationary points of the function

$$g(x, y) = x^3 + xy^2 - 12x - y^2$$

Classify the nature of the stationary points and calculate the value of g at these points.

(18 marks)

(b) Use Lagrange multipliers to find the temperature of the hottest and coldest points on a circle (centered on the origin) of radius 2, where the temperature is given by

$$T(x, y) = 1 + xy.$$

(15 marks)

3. (a) The function $f(x)$ is defined on the interval $-2 \le x \le 2$ by

$$f(x) = x + |x|,$$

while $f(x) = f(x-4)$ for $x > 2$ and $f(x) = f(x+4)$ for $x < -2$.

   i. Sketch the function on the interval $[-8, 8]$.

   ii. Determine the Fourier Series for this function.

(20 marks)

(b) Prove that if a periodic function f (of period $2L$) is odd, then the Fourier coefficients

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right) dx$$

are all zero. (13 marks)

4. (a) Evaluate

$$\int\int (x^2 + y^2) dx dy$$

over the triangle with vertices (0,0), (2,0) and (1,1). (11 marks)

(b) Calculate

$$\int_0^2 \int_{x^2}^4 x^3 y dy dx.$$

(11 marks)

(c) Use polar coordinates to evaluate the integral

$$\int\int_A e^{-(x^2+y^2)} dx dy$$

where A is the disc given by

$$3 \leq x^2 + y^2 \leq 4.$$

(11 marks)

5. Verify Green's theorem for the integral

$$\oint_{\partial C} (x + y) dx + x^2 dy$$

where C is bounded by the x and y axes and the line $y = 1 - x/2$. (33 marks)