Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX ______

Semester II Examinations, 2003/2004

Exam Code(s)	1MS1, 1SD1
Exam(s)	MS1 M.Sc.Degree
	SD1 H.Dip.(Software Design And Development)
Module Code(s)	CT853
Module(s)	Algorithmics & Logical Methods
Paper No.	1
Repeat Paper	______ Special Paper ______
External Examiner(s)	Professor D. Bell
Internal Examiner(s)	Professor G. Lyons
	Dr. M. Mc Gettrick

Instructions Answer 3 questions.
All questions will be marked equally.

Duration	2hrs
No. of Answer Books	1

Requirements

Handout ______
MCQ ______
Statistical Tables ______
Graph Paper ______
Log Graph Paper ______
Other Material ______

No. of Pages ______
Department(s) ______

1. (a) Using each of the following methods, write down (step by step) the position of each digit in the number 28715 when sorted using
   (i) insertion sort
   (ii) quicksort

   (b) State (in "Big Oh" notation) the worst case complexity of quicksort and the best case complexity of insertion sort. Assuming average case complexity, suppose it takes t_1 seconds for a quicksort of a million items. Calculate (in terms of t_1) how long it would take to sort eight million items using quicksort.

2. (a) Write (using pseudocode) an algorithm that uses a Binary Search to find an object X in an ordered list L that has n objects $L(1), L(2), L(3), \ldots L(n)$. Assume X occurs at most once: If X is not in L your algorithm should state so; If it is, your algorithm should state where it is located (i.e. the index i where $L(i) = X$).

   (b) A **Mersenne Number** is one of the form $2^n - 1$, where n is a positive integer. The first few numbers are 1 $(2^1 - 1)$, 3 $(2^2 - 1)$, 7 $(2^3 - 1)$, etc. Write (in pseudocode) an $\mathrm{O}(\log(n))$ algorithm using the idea of Binary Powering to calculate the nth Mersenne Number.

3. (a) Explain what is meant by **recursion**.

   (b) The **Fibonacci Numbers**

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

   are defined by $f_0 = 0, f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n > 1$. Write two different algorithms to calculate the nth Fibonacci number, one of which uses recursion. State (by comparing the complexity of both algorithms) which is more efficient.

4. (a) Use truth tables to determine whether each of the following well formed formulae (wff) are tautologies, contradictions, or neither.
   (i) $(p \wedge \neg p) \rightarrow (q \wedge \neg q)$
   (ii) $\neg[(p \rightarrow q) \wedge (q \rightarrow r)] \vee (p \rightarrow r)$
   (iii) $[\neg p \wedge (p \vee q)] \wedge \neg q$

   (b) Given the predicates

$$\mathrm{W(x)} : \text{“x is a wind instrument” (Universe } U \equiv \text{ set of all musical instruments, } x \in U)$$
$$\mathrm{I(y)} : \text{“y is Italian” (Universe } V \equiv \text{ set of all people, } y \in V)$$
$$\mathrm{P(p,q)} : \text{“p plays (musical instrument) q” } (p \in V, q \in U)$$

   write statements in First Order Predicate Calculus to express each of the following:
   (i) Helena does not play the Bassoon.
   (ii) Either all instruments are wind instruments, or only Italians play the piano.
   (iii) No wind instrument is played by everyone.
   (iv) Some non-Italians play all the wind instruments.