Ollscoil na hÉireann, Gaillimh GX ______
National University of Ireland, Galway

Semester II Examinations, 2003/2004

Exam Code(s)	0MS1, 1MF1
Exam(s)	MS1 M.Sc.Degree
	MF1 M.Sc.Degree(Software Design & Development)
Module Code(s)	CT518
Module(s)	Algorithms And Logical Methods
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Professor D. Bell
Internal Examiner(s)	Professor G. Lyons
	Dr. M. Mc Gettrick

Instructions Answer 4 questions.
All questions will be marked equally.

Duration	2hrs
No. of Answer Books	1

Requirements

Handout
MCQ
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

No. of Pages
Department(s)

1. (a) Using each of the following methods, write down (step by step) the position of each digit in the number 28715 when sorted using
   (i) insertion sort
   (ii) quicksort
   (b) State (in "Big Oh" notation) the worst case complexity of quicksort and the best case complexity of insertion sort. Assuming average case complexity, suppose it takes t_1 seconds for a quicksort of a million items. Calculate (in terms of t_1) how long it would take to sort eight million items using quicksort.

2. (a) Write (using pseudocode) an algorithm that uses a Binary Search to find an object X in an ordered list L that has n objects $L(1), L(2), L(3), \ldots L(n)$. Assume X occurs at most once: If X is not in L your algorithm should state so; If it is, your algorithm should state where it is located (i.e. the index i where $L(i) = X$).
   (b) A **Mersenne Number** is one of the form $2^n - 1$, where n is a positive integer. The first few numbers are 1 $(2^1 - 1)$, 3 $(2^2 - 1)$, 7 $(2^3 - 1)$, etc. Write (in pseudocode) an $O(\log(n))$ algorithm using the idea of Binary Powering to calculate the nth Mersenne Number.

3. (a) Explain what is meant by **recursion**.
   (b) The **Fibonacci Numbers**

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

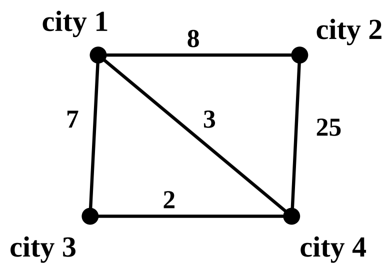
   are defined by $f_0 = 0, f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for $n > 1$. Write two different algorithms to calculate the nth Fibonacci number, one of which uses recursion. State (by comparing the complexity of both algorithms) which is more efficient.

4. (a) Use truth tables to determine whether each of the following well formed formulae (wff) are tautologies, contradictions, or neither.
   (i) $(p \wedge \neg p) \rightarrow (q \wedge \neg q)$
   (ii) $\neg[(p \rightarrow q) \wedge (q \rightarrow r)] \vee (p \rightarrow r)$
   (iii) $[\neg p \wedge (p \vee q)] \wedge \neg q$
   (b) Given the predicates

   W(x) : "x is a wind instrument" (Universe $U \equiv$ set of all musical instruments, $x \in U$)
   I(y) : "y is Italian" (Universe $V \equiv$ set of all people, $y \in V$)
   P(p,q) : "p plays (musical instrument) q" ($p \in V, q \in U$)

   write statements in First Order Predicate Calculus to express each of the following:
   (i) Helena does not play the Bassoon.
   (ii) Either all instruments are wind instruments, or only Italians play the piano.
   (iii) No wind instrument is played by everyone.
   (iv) Some non-Italians play all the wind instruments.

5. (a) The diagram below shows a road network between four cities, with the length of each road marked. Let D_{ij} be the length of the shortest route from city i to city j (e.g. D_{13} = length of shortest route from city 1 to city 3 = 5 (travelling via city 4)) **Using the Dynamic Programming method**, calculate the matrix D (Hint: start by writing down the matrix A where A_{ij} is the length of the edge between i and j, $A_{ij} = \infty$ if there is no edge between i and j, and $A_{ii} = \infty$). For full marks you must both obtain the correct answer **and** illustrate your method.



(b) Explain by giving an example each of the following:

(i) Idempotency of $\vee$

(ii) The Law of Double Negation

(iii) Distribution Law of $\wedge$ over $\vee$

(iv) Commutativity of $\vee$