Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX ______

Semester II Examinations, 2006/2007

Exam Code(s)	2IF1, 3EL2
Exam(s)	IF1 B.Sc. (Information Technology)
	EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)
Module Code(s)	CT214
Module(s)	Logical Foundations Of Computing
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Dr. J. A. Keane, Dr. D. Johnson
Internal Examiner(s)	Professor G. Lyons, Professor J. Hinde
	Dr. E. Sköldberg
	Dr. M. Mc Gettrick

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	3hrs
No. of Answer Books	2

Requirements

Handout
MCQ
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

No. of Pages
Department(s)

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

A1. (i) Explain the meaning of the terms *tautology*, *contradiction*, *contingency* and *logical equivalence.*

(ii) For each of the following propositions, give its truth table and hence determine if it is a tautology, contradiction or contingency:

(a) $p \to (\neg p \to q)$,

(b) $p \wedge (p \to \neg q) \wedge q$,

(c) $\perp \to p$.

(iii) For each of the following propositions, find a disjunctive normal form:

(a) $(p \vee q) \leftrightarrow (p \wedge r)$,

(b) $\neg(\neg p \to \neg q)$.

A2. (i) (a) What is a valuation?

(b) Show that the equality $[\![\phi \vee \psi]\!]_v = [\![\phi]\!]_v + [\![\psi]\!]_v - [\![\phi]\!]_v \cdot [\![\psi]\!]_v$ holds for all valuations v.

(ii) Using the method of truth tables, show that the following logical equivalences hold:

(a) $\phi \wedge (\psi \vee \sigma) \equiv (\phi \wedge \psi) \vee (\phi \wedge \sigma)$,

(b) $\phi \vee (\neg \phi \wedge \psi) \equiv \phi \vee \psi$,

(c) $\phi \to \psi \equiv \neg \phi \vee \psi$.

(iii) Use the method of resolution to determine if the following arguments are valid:

(a) $\{p \to q, \neg r \to \neg q, p \vee q\} \models r$,

(b) $\{p \leftrightarrow q, r \leftrightarrow s, p \to r\} \models q \to s$.

A3. (i) For each of the following propositions, determine wheter it is in CNF, DNF, both or neither:

(a) $(p \wedge \neg q) \vee (r \wedge \neg s)$,

(b) $(p \wedge q) \vee r \vee s$,

(c) $p \wedge q$.

(ii) Show the following using natural deduction:

(a) $\vdash (p \to q) \wedge (q \to r) \to (p \to r)$,

(b) $\vdash p \to (q \to p)$,

(c) $\{\neg p, p \vee q\} \vdash q$.

SECTION B
Answer Only 2 Questions

B1. (i) Consider a generic binary operator $\Box$ on some set S. Explain what is meant by saying $\Box$ is

(a) idempotent
(b) commutative
(c) associative

Prove that if all three properties hold then $a\Box(b\Box a) = b\Box a \quad \forall a, b \in S$

(ii) Using the laws of Propositional Calculus, prove the following:

(a) $(a \vee b) \vee (c \vee d) = a \vee ((b \vee c) \vee d)$
(b) $a \rightarrow (b \rightarrow c) = b \rightarrow (a \rightarrow c)$
(c) $p \rightarrow (q \rightarrow r) = (p \rightarrow q) \rightarrow (p \rightarrow r)$
(d) $((p \vee q) \wedge \neg p) \rightarrow q = ((p \rightarrow r) \wedge p) \rightarrow r$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

B2. (i) On the set $\mathbb{Z}$ of all integers, consider the predicates $p_1(x,y) : x = y, \quad p_2(x) : x > 1, \quad p_3(x,y,z) : x + y = z$, where $x, y, z \in \mathbb{Z}$. State whether each of the following is true or false:

(a) $\forall x : \mathbb{Z} \bullet p_2(x)$
(b) $\forall x : \mathbb{Z} \bullet \forall y : \mathbb{Z} \bullet p_1(x,y)$
(c) $\exists x : \mathbb{Z} \bullet \exists y : \mathbb{Z} \bullet p_3(x,y,7)$
(d) $\forall x : \mathbb{Z} \bullet \forall y : \mathbb{Z} \bullet \forall z : \mathbb{Z} \bullet (p_2(x) \wedge p_1(x,y) \wedge p_3(x,y,z)) \rightarrow p_2(z)$
(e) $\exists x : \mathbb{Z} \bullet \forall y : \mathbb{Z} \bullet p_3(x,-2,y)$
(f) $\forall x : \mathbb{Z} \bullet \exists y : \mathbb{Z} \bullet p_3(8,x,y)$

(ii) Let U be the Universe of people ($x, y \in U$) on which we define the following Atomic Predicates:
$F(x,y)$: x is a friend of y
$M(x,y)$: x is married to y
$Y(x,y)$: x is younger than y
Represent the following statements in Predicate Calculus:

(a) Everyone has a friend.
(b) Everyone has a younger friend who is not married.
(c) Bob is older than all the married people.
(d) Nobody who is married is a friend of everyone else.

B3. (i) Consider the following argument:

"If it rains a lot in the Spring, the apple trees or the pear trees will grow well. If the apple trees grow well and the pear trees don't, there will be a large number of insects in the Autumn. The pear trees don't grow well. Therefore, if it rains a lot in the Spring, there will be a large number of insects in the Autumn"

(a) Using Propositional Calculus, prove this is a valid argument, stating the laws you use at each step of the proof.

(b) Write two alternative proofs, one using the Deduction Theorem, and the other using Reductio ad Absurdum.

(ii) Prove that

(a) Modus Tollens

(b) Hypothetical Syllogism

are valid inference rules.