Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX ______

Semester II Examinations, 2005/2006

Exam Code(s)	2IF1, 3EL2
Exam(s)	IF1 B.Sc. (Information Technology)
	EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)
Module Code(s)	CT214
Module(s)	Logical Foundations Of Computing
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Dr. J. A. Keane, Dr. D. Johnson
Internal Examiner(s)	Dr. J. Duggan, Professor J. Hinde
	Dr. E. Sköldberg
	Dr. M. Mc Gettrick

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	3hrs
No. of Answer Books	2

Requirements

Handout
MCQ
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

No. of Pages
Department(s)

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

A1. (i) Explain the meaning of the terms *tautology*, *contradiction*, *contingency* and *logical equivalence.*

(ii) For each of the following propositions, give its truth table, and hence or otherwise determine if it is a tautology, contradiction or neither.

(a) $((p \vee q) \to p) \to q$

(b) $\neg(p \wedge q) \leftrightarrow (p \to \neg q)$

(c) $(p \wedge q) \wedge (\neg p \vee \neg q)$

(d) $p \to (p \wedge \neg p)$

A2. (i) Use the method of resolution to check if the following arguments are valid:

(a) $\{\neg p \to q, \neg q \vee \neg r, \neg p \to r\} \models p$

(b) $\{p \vee q \vee r, (p \vee q) \to s, (q \wedge r) \to s\} \models s$

(c) $\{p \to q, q \to r, r \to s, \neg p\} \models \neg s$

(ii) State the completeness theorem of propositional logic, and by using it or otherwise show the following

(a) $\vdash (p \wedge q) \to (q \wedge p)$

(b) $\vdash p \to (p \vee q)$

A3. (i) Find the disjunctive normal form of the following propositions.

(a) $(p \vee q) \leftrightarrow (r \to q)$

(b) $\neg(\neg p \vee \neg q)$

(c) $(p \vee p) \to p$

(ii) Using natural deduction, show the following.

(a) $\vdash p \to (q \to (p \wedge q))$

(b) $\vdash (p \vee q) \to (q \vee p)$

(c) $\vdash p \to \neg\neg p$

(d) $\vdash (p \to q) \to (\neg q \to \neg p)$

SECTION B
Answer Only 2 Questions

B1. (i) Consider the equivalence (biconditional) connective $\leftrightarrow$:

(a) Is it idempotent?

(b) Does it have an identity element (if so state the element)?

(c) Does it have an absorption element (if so state the element)?

(d) Write down the law of the complement for $\leftrightarrow$.

(ii) Using the laws of Propositional Calculus, prove the following:

(a) $(p \wedge (q \rightarrow r)) \vee \neg p = q \rightarrow (p \rightarrow r)$

(b) $a \rightarrow (b \rightarrow (\neg a \rightarrow c)) = d \wedge \neg d$

(c) $p \wedge \neg(p \wedge q) = \neg(p \rightarrow q)$

(d) $p \wedge [\neg p \vee (p \rightarrow \neg p)] = q \leftrightarrow \neg q$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

B2. (i) Consider the following argument:

"If it rains a lot in the Spring, the apple trees and pear trees will grow well. If either the apple trees or the pear trees grow well, there will be a large number of insects in the Autumn. Therefore, if it rains a lot in the Spring, there will be a large number of insects in the Autumn"

(a) Using Propositional Calculus, show whether or not the argument is valid, stating the laws you use.

(b) Write two alternative proofs, one using the Deduction Theorem, and the other using Reductio ad Absurdum.

(ii) Prove that

(a) Constructive Dilemma

(b) Disjunctive Syllogism

are valid inference rules.

B3. (i) Let $R = \{x \in S | R(x) = 1\}$ and $P = \{x \in S | P(x) = 1\}$ where R and P are functions from S to the Boolean set $B = \{0, 1\}$. Draw Venn diagrams to illustrate the statements

(a) "All R are P"

(b) "Some R are P"

(c) "Some P are R"

(d) "No R is P"

(e) "All R are not P"

Are any of these pictures the same?

(ii) Let U_1 be the Universe of people ($x, y \in U_1$) and U_2 be the Universe of languages ($p, q \in U_2$) on which we define the following Atomic Predicates:

$N(x, y)$: x and y have the same nationality

$S(x, y, p)$: x speaks to y in language p

$I(x)$: x is Irish

$P(x)$: x is Polish

Represent the following statements in Predicate Calculus:

(a) Some Irish people speak to one another in the Irish language.

(b) People who are not of the same nationality always speak English to one another.

(c) Irish people never speak the Irish language to Polish people.

(d) If three people have the same nationality, then none of them is Irish.