Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX ______

Semester II Examinations, 2004/2005

Exam Code(s)	2IF1, 3EL2
Exam(s)	IF1 B.Sc. (Information Technology)
	EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)
Module Code(s)	CT214
Module(s)	Logical Foundations Of Computing
Paper No.	1
Repeat Paper	______ Special Paper ______
External Examiner(s)	Dr. Dave Johnson, Professor P. Nixon
Internal Examiner(s)	Professor G. Lyons, Professor T. Hurley
	Dr. E. Sköldberg
	Dr. M. Mc Gettrick

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	3hrs
No. of Answer Books	2

Requirements

Handout	
MCQ	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	

No. of Pages	
Department(s)	

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

A1. (i) Define what it means for two propositions to be *logically equivalent.*

(ii) For each of the following propositions, give its truth table, and hence determine if it is a tautology, contradiction or neither.

(a) $\neg(\neg p \vee \neg q) \leftrightarrow (p \wedge q)$

(b) $(p \wedge q) \wedge (\neg p \vee \neg q)$

(c) $\neg(\neg(p \vee \neg p))$

(d) $((p \wedge q) \rightarrow r) \rightarrow (p \rightarrow r)$

A2. (i) Give the truth tables for $\phi \rightarrow \psi$, and $\phi \leftrightarrow \psi$.

(ii) Use the method of resolution to check if the following arguments are valid:

(a) $\{p \rightarrow q, q \rightarrow r, r \rightarrow s, \neg s\} \models \neg p$

(b) $\{p \vee q, q \vee r, q \rightarrow \neg s, p \wedge r\} \models \neg s$

(c) $\{p \rightarrow (q \vee r), \neg r \rightarrow \neg q, r\} \models p$

A3. (i) Find the disjunctive normal form of the following propositions.

(a) $(p \wedge (\neg q \leftrightarrow r)) \rightarrow r$

(b) $p \rightarrow (q \wedge r)$

(c) $(\neg p \leftrightarrow q) \wedge (p \vee q)$

(ii) Using natural deduction, show the following.

(a) $\vdash (\neg q \rightarrow \neg p) \rightarrow (p \rightarrow q)$.

(b) $\vdash \neg(\neg p \wedge p)$

(c) $\vdash p \rightarrow (\neg p \rightarrow q)$.

SECTION B
Answer Only 2 Questions

B1. (i) Give an example of each of the following:

(a) Associativity of $\vee$

(b) Commutativity of $\wedge$

(c) Absorption element of $\wedge$

(d) Distribution of $\wedge$ over $\vee$

(ii) Using the laws of Propositional Calculus, prove the following:

(a) $p \rightarrow (q \rightarrow r) = q \rightarrow (p \rightarrow r)$

(b) $p \wedge \neg(p \wedge q) = p \wedge \neg q$

(c) $((q \vee p) \vee r) \vee s = p \vee (q \vee (s \vee r))$

(d) $(p \rightarrow \neg q) \leftarrow r = (q \wedge r) \rightarrow \neg p$

(e) $[p \wedge (q \vee r)] \wedge \neg q = (p \wedge \neg q) \wedge r$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

B2. (i) Consider the implication $p \to q$. Write down (a) the antecedent, (b) the conclusion, (c) the converse, and (d) the contrapositive of this statement.

(ii) Define $p \equiv q$ to mean $(p \wedge q) \vee (\neg p \wedge \neg q)$. Prove, stating the laws you use at each step, that an equivalent definition is $p \equiv q = (p \to q) \wedge (q \to p)$.

(iii) Consider the following statements:

`S1`: The Sun is in orbit around the Earth.

`S2`: At most one of these statements is true (i.e. at least one of `S1` or `S2` is false!).

Can a consistent set of truth values be assigned to `S1` and `S2`? If so, state the assignment(s) and the resulting value(s) of `S1`.

B3. (i) Consider the following argument:
"If Ireland beat Germany then they will beat Japan. Ireland beat Germany and Ireland win the World Cup. Ireland don't beat Japan or Ireland don't win the World Cup. Therefore the Earth is Flat"

(a) Using Propositional Calculus, show whether or not the argument is valid, stating the laws you use.

(b) Explain why the premises of the argument allow one to reach such a strange conclusion.

(ii) Let U be the Universe of people ($x, y \in U$) on which we define the following Atomic Predicates:
$F(x, y) : x$ is a friend of y
$M(x, y) : x$ is married to y
$Y(x, y) : x$ is younger than y
Represent the following statements in Predicate Calculus:

(a) Everyone has a friend.

(b) Everyone has a married friend who is older than themselves.

(c) Bob is unmarried and has no friends.

(d) The oldest person is a friend of everyone.