Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX

Semester II Examinations, 2003/2004

Exam Code(s)	2IF1, 3EL2
Exam(s)	IF1 B.Sc. (Information Technology)
	EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)
Module Code(s)	CT214
Module(s)	Logical Foundations Of Computing
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Dr. Dave Johnson, Professor P. Nixon
Internal Examiner(s)	Professor G. Lyons, Professor T. Hurley
	Dr. E. Sköldberg
	Dr. M. Mc Gettrick

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	3hrs
No. of Answer Books	2

Requirements

Handout
MCQ
Statistical Tables
Graph Paper
Log Graph Paper
Other Material

No. of Pages
Department(s)

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

A1. (i) Define what it means for two propositions to be *logically equivalent.*

(ii) Define what it means that a logical proposition is a *tautology* and a *contradiction*

(iii) For each of the following propositions, determine if it is a tautology, a contradiction or neither.

(a) $\neg(p \to \neg p)$

(b) $(\neg p \vee q) \leftrightarrow (p \to q)$

(c) $(p \wedge q) \vee (\neg p \wedge \neg q)$

A2. (i) Write down the truth tables for $p \vee q$, $p \wedge q$ and $\neg p$

(ii) Use the method of resolution to check if the following arguments are valid:

(a) $\{p \vee q, p \to s, q \to s\} \models s$

(b) $\{q, \neg p \leftrightarrow \neg q\} \models p$

A3. (i) Find the disjunctive normal form of the following propositions

(a) $\neg p \wedge (\neg q \to r)$

(b) $p \leftrightarrow (q \wedge r)$

(c) $\neg(p \to q)$

(ii) Using natural deduction, show the following

(a) $\vdash (\neg p \to p) \to p$

(b) $\vdash (p \to q) \to (\neg q \to \neg p)$

SECTION B
Answer Only 2 Questions

B1. (i) Give an example of each of the following:

(a) Idempotency of $\vee$

(b) Left Identity element of $\to$

(c) Absorption element of $\wedge$

(d) Distribution of $\vee$ over $\wedge$

(ii) Using the laws of Propositional Calculus, prove the following:

(a) $p \to (q \to r) = q \to (p \to r)$

(b) $(p \vee q) \to (\neg p \wedge q) = \neg p$

(c) $((q \vee p) \vee r) \vee s = p \vee (q \vee (s \vee r))$

(d) $(p \to \neg q) \leftarrow r = (q \wedge r) \to \neg p$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(iii) Prove that *Hypothetical Syllogism* is a valid inference rule.

B2. (i) Construct two separate proofs using (a) the Deduction Theorem (b) Reductio ad Absurdum for each of the following (in each case give a reason for every step):

(a)

$$\{p \rightarrow q,\ r \rightarrow s,\ p \vee r\} \vdash \neg q \rightarrow s$$

(b) The argument:

If it rains on Monday it will be sunny and dry on Tuesday. If it is sunny or dry on Tuesday, then it will be cloudy on Wednesday. Therefore, if it rains on Monday it will be cloudy on Wednesday.

(ii) Consider the following paradox:

`S1`: The Sun is in orbit around the Earth.

`S2`: Neither of these statements is true (i.e. neither `S1` nor `S2` is true).

Can a consistent set of truth values be assigned to `S1` and `S2`? If so, state the assignment(s) and the resulting value(s) of `S1`.

B3. (i) For each of the expressions

(i) $\exists x : U_1 \bullet \forall y : U_2 \bullet P(x, y)$

(ii) $\forall x : U_1 \bullet \exists y : U_2 \bullet P(x, y)$

determine the resulting truth value when

(a) $U_1 = \phi \neq U_2$

(b) $U_2 = \phi \neq U_1$

(c) $U_1 = \phi = U_2$

(ϕ is the empty or null set $\{\}$)

(ii) Let U be the set of all people ($x \in U$) and V be the set of all languages ($y \in V$). We define the following Atomic Predicates:

$S(x, y)$: person x speaks language y

$K(x)$: person x is Korean

$IE(y)$: y is an Indo-European language

Represent the following statements in Predicate Calculus:

(a) Korean people speak some Indo-European languages.

(b) All Indo-European languages are spoken by somebody.

(c) If everyone speaks every language, then no one speaks French.

(d) The only people who speak Korean are Korean.

(e) Non-Koreans do not speak all Indo-European languages.