OLLSCOIL NA hÉIREANN GAILLIMH

NATIONAL UNIVERSITY OF IRELAND GALWAY

SEMESTER II
SUMMER EXAMINATIONS 2001/2002

Second University Examination in Information Technology

LOGICAL FOUNDATIONS OF COMPUTING (CT214)

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Time allowed: *three* hours.
Candidates should attempt four questions, two from each section.
Please use separate answer books for each section.
All questions carry equal marks.

SECTION A

A1. (i) Write down the logic table for the gate $a \to b$ (*if* a *then* b).
Show that $(a \to b)$ is logically equivalent to $(\overline{a} \vee b)$.
Give an equivalent Boolean expression that uses only **NOT** and **AND**.

(ii) Construct the logic table for the Boolean expression

$$(x_1 \to \overline{x}_2) \to \Big((x_1 \vee x_2) \wedge \overline{x}_3\Big).$$

Write down the corresponding disjunctive normal form (DNF).

(iii) What is a **Basic Machine**?
"A certain written exam consists of 3 sections. To pass the exam, a student must pass at least two of the sections".
Construct a basic machine to model this and explain the notation used.

P.T.O.

A2. (i) When is a collection of (compound) propositions **inconsistent**?

For each of the following, determine if it is consistent or inconsistent:

(a) $\overline{a} \wedge \overline{b}, a \vee \overline{b}, a \rightarrow b$

(b) $\overline{a} \wedge b, a \vee \overline{b}, a \rightarrow b$

(ii) What is the definition of a **valid argument**?

Prove that the argument $P_1, P_2 \therefore C$ is valid if and only if $P_1, P_2, \sim C$ are inconsistent as a collection.

Use this to show that

$$a \vee b, a \rightarrow b \therefore b$$

is a valid argument.

(iii) A logician reasons as follows:

"If the argument is valid and the premises are true then the conclusion is true.
The premises are false but the conclusion is true.
So the argument is valid."

Using a, b and c for "the argument is valid", "the premises are true" and "the conclusion is true", construct the argument which underlies the above reasoning. Test the validity of the argument you have constructed.

A3. (i) What is a *Well-Formed Formula* (WFF)? Using two boolean variables, a and b, and the **AND**, **OR** and **NOT** gates, give an example of an expression that

(a) **is not** a WWF,

(b) **is** a WWF.

(ii) Use the *Semantic Tableau* method to show that the following collection of WFFs is consistent:

$$a \rightarrow (b \vee c), b \rightarrow a, \sim c.$$

For each open branch of the tableau, read off the assignment of values to a, b and c which makes all three WFFs take the value 1.

(iii) Give a brief description of the *Resolution Method.*

Use resolution to show that the following argument is valid:

$$\overline{a} \vee (b \wedge c), b \rightarrow (a \wedge \overline{c}) \therefore \overline{a}$$

SECTION B

B1. (i) Using the laws of Propositional Calculus, prove the following:

(a) $(p \wedge q) \rightarrow r = p \rightarrow (q \rightarrow r)$

(b) $[p \wedge (q \vee r)] \wedge \neg q = (p \wedge \neg q) \wedge r$

(c) $p \wedge [(p \wedge q) \vee r] = p \wedge (q \vee r)$

(d) $p \wedge \neg(p \wedge q) = p \wedge \neg q$

Give a reason for each step, and if you combine several steps into one, list all the laws used.

(ii) Prove (stating the laws you use at each step) that Modus Tollens is a Valid Inference Rule, i.e. that $([(p \rightarrow q) \wedge \neg q] \rightarrow \neg p)$ is a tautology.

(iii) Explain how (a) the Deduction Theorem and (b) Reductio ad Absurdum can be used to construct proofs.

P.T.O.

B2. (i) Consider the implication $p \rightarrow q$. Write down (a) the antecedent, (b) the conclusion, (c) the converse, and (d) the contrapositive of this statement.

(ii) Define $p \equiv q$ to mean $(p \wedge q) \vee (\neg p \wedge \neg q)$. Prove, stating the laws you use at each step, that an equivalent definition is $p \equiv q = (p \rightarrow q) \wedge (q \rightarrow p)$.

(iii) Using Propositional Calculus, show whether or not the following argument is valid, stating the laws you use:
"If the weather is cool and Ireland beat Germany then Ireland will win the World Cup. The weather is cool, but Ireland do not win the World Cup. So, Ireland did not beat Germany."

B3. (i) Consider the following argument:
"If Ireland beat Germany then they will beat Japan. Ireland beat Germany and Ireland win the World Cup. Ireland don't beat Japan or Ireland don't win the World Cup. Therefore the Earth is Flat"

(a) Using Propositional Calculus, show whether or not the argument is valid, stating the laws you use.

(b) Explain why the premises of the argument allow one to reach such a strange conclusion.

(ii) Assuming the general De Morgan Law from propositional Calculus ($\neg(p_1 \wedge p_2 \wedge p_3 \wedge ... \wedge p_n) = \neg p_1 \vee \neg p_2 \vee \neg p_3 \vee ... \vee \neg p_n$) prove the De Morgan law for relative quantification in a finite Universe U:

$$\neg[\forall x : U \bullet P(x) \Rightarrow Q(x)] = \exists x : U \bullet P(x) \wedge \neg Q(x)$$

Give an example to show why the following is not valid:

$$\forall x : U_1 \bullet \exists y : U_2 \bullet P(x, y) = \exists y : U_2 \bullet \forall x : U_1 \bullet P(x, y)$$

(iii) Let U be the Universe of people $(x, y \in U)$ on which we define the following Atomic Predicates:
$F(x, y)$: x is a friend of y
$M(x, y)$: x is married to y
$Y(x, y)$: x is younger than y
Represent the following statements in Predicate Calculus:

(a) Everyone has a friend.

(b) Everyone has a married friend who is older than themselves.

(c) Bob is unmarried and has no friends.

(d) Nobody who is married is younger than everyone else.