

Tangents

Example 1.1

Determine the (equation of the) tangent (line) to the function

$$f(x) = x^2$$

at the point $x = 5$.

$$f(x) = x^2 \implies f'(x) = 2x$$

$$\text{At the point } x = 5, y = f(x) = f(5) = 5^2 = 25. \quad (1)$$

$$\text{At the point } x = 5, f'(x) = f'(5) = (2)(5) = 10.$$

This means, the tangent line has slope 10 and goes through the point $(5, 25)$. The equation of a line with slope m is $y = mx + c$. So here we have $y = 10x + c$. Since the line must go through $(5, 25)$, we insert this point in to the equation to get $25 = (10)(5) + c \implies c = -25$. Finally our answer is $y = 10x - 25$.

Example 1.2

Find the points on the curve $y = 4x^3 - 6x$ where the tangent (line) is parallel to $2x + y + 5 = 0$.

To say two lines are “parallel” means they have the same slope:

- 1 What is the slope of the line? Rewrite it as $y = -2x - 5$, so the slope is -2 . (The slope of a line $y = mx + c$ is m .)
- 2 What is the slope at any point on the curve?
 $y = 4x^3 - 6x \implies y' = 12x^2 - 6$.

So we just set these two slopes equal to one another:

$$-2 = 12x^2 - 6 \implies 4 = 12x^2 \implies x = 1/\sqrt{3} \text{ or } x = -1/\sqrt{3}.$$

- $x = 1/\sqrt{3} \implies y = 4(1/\sqrt{3})^3 - 6/\sqrt{3} = -14/(3\sqrt{3})$
- $x = -1/\sqrt{3} \implies y = -4(1/\sqrt{3})^3 + 6/\sqrt{3} = 14/(3\sqrt{3})$