

MA140-Engineering Calculus

Lecture 3

September 7, 2017

1. Linear factors to the power of 1.

Example 1.1

Express $\frac{3x}{(x-1)(x+2)}$ in partial fractions.

$$\begin{aligned}\frac{3x}{(x-1)(x+2)} &= \frac{A}{x-1} + \frac{B}{x+2} \\ &= \frac{A(x+2) + B(x-1)}{(x-1)(x+2)} \\ &= \frac{x[A+B] + [2A-B]}{(x-1)(x+2)}\end{aligned}$$

Compare $x[3] + [0]$ with $x[A + B] + [2A - B]$, so

$$\begin{cases} A + B = 3 \\ 2A - B = 0 \end{cases}$$

So $3 = 3A \Rightarrow A = 1$

From first equation $3 = 1 + B \Rightarrow B = 2$

so

$$\frac{3x}{(x-1)(x+2)} = \frac{1}{x-1} + \frac{2}{x+2}$$

Exercise:

Find the constants A , B and C , so that:

$$\frac{2x + 1}{(x-2)(x+1)(x-3)} = \frac{A}{x-2} + \frac{B}{x+1} + \frac{C}{x-3}$$

2. Linear Factors to Powers Greater than 1 (i.e. repeated linear factors):

If $(x - \alpha)^k$ appears in the denominator, it will give rise to the following terms:

$$\frac{A_1}{x - \alpha} + \frac{A_2}{(x - \alpha)^2} + \cdots + \frac{A_k}{(x - \alpha)^k}$$

Exercise:

Show from the start

$$\frac{3x + 1}{(x - 1)^2(x + 2)} = \frac{5/9}{x - 1} + \frac{4/3}{(x - 1)^2} + \frac{-5/9}{x + 2}$$

3. Irreducible quadratic factors:

Irreducible quadratic factors can not be factorised using real numbers e.g.

$$x^2 + x + 1$$

An irreducible quadratic $ax^2 + bx + c$ gives rise to partial fractions of the form

$$\frac{Ax + B}{ax^2 + bx + c}$$

Example 1.2

Express the following in partial fractions:

$$\frac{5x}{(x^2 + x + 1)(x - 2)}$$

$$\begin{aligned} \frac{5x}{(x^2 + x + 1)(x - 2)} &= \frac{Ax + B}{x^2 + x + 1} + \frac{C}{x - 2} \\ &= \frac{(Ax + B)(x - 2) + C(x^2 + x + 1)}{(x^2 + x + 1)(x - 2)} \end{aligned}$$

Therefore: $5x = (Ax + B)(x - 2) + C(x^2 + x + 1)$

Now let $x = 2$ then

$$5(2) = 0 + C(2^2 + 2 + 1)$$

$$\Rightarrow 10 = C(7)$$

$$\Rightarrow C = 10/7$$

constant term on the RHS=0 so

$$0 = -2B + C$$

$$\Rightarrow 2B = C$$

$$\Rightarrow B = C/2 = 5/7$$

The coefficient of $x^2 = 0$ so $0 = A + C \Rightarrow A = -C = -10/7$, so

$$\frac{5x}{(x^2 + x + 1)(x - 2)} = \frac{-\frac{10}{7}x + \frac{5}{7}}{x^2 + x + 1} + \frac{\frac{10}{7}}{x - 2}$$

4. Irreducible quadratic factors to powers greater than 1:

If $ax^2 + bx + c$ is an irreducible quadratic factor, then $(ax^2 + bx + c)^k$ in the denominator gives rise to

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_kx + B_k}{(ax^2 + bx + c)^k}$$