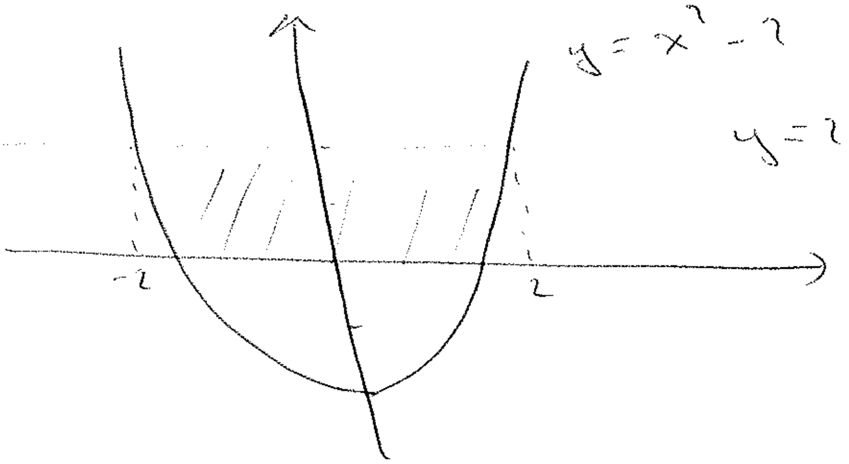
1

Find the area of the region enclosed by the line and the curve:

$y = x^2 - 2$ and $y = 2$

ans:



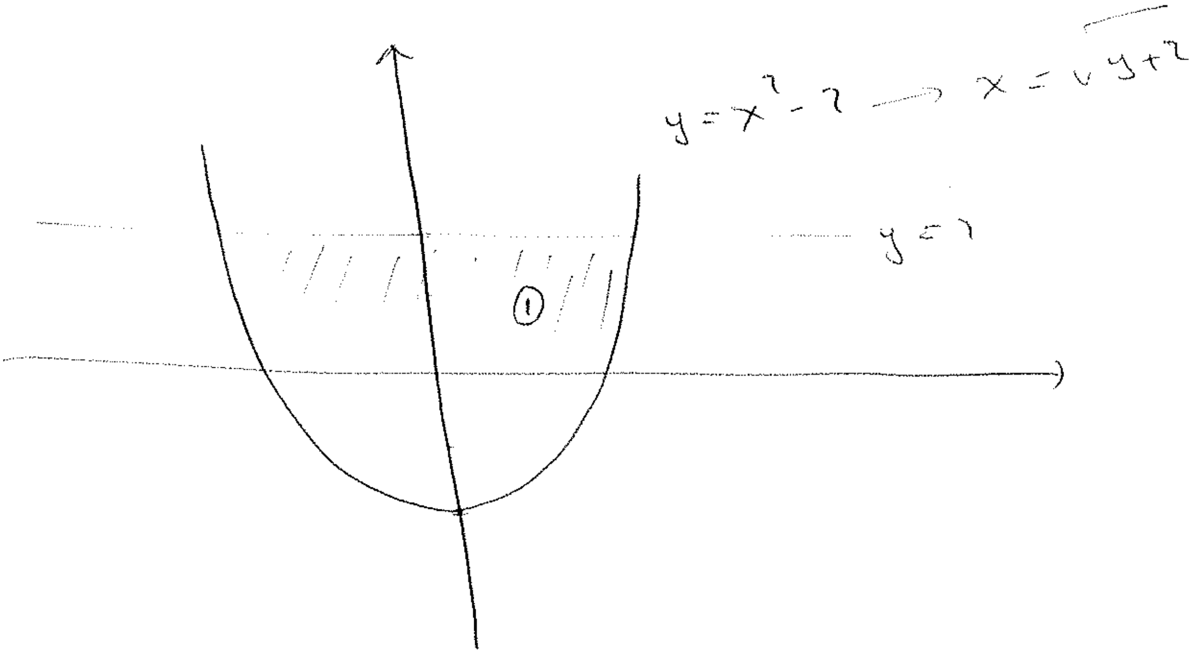
$$\text{area} = \int_{-2}^{2} (\text{upper curve} - \text{lower curve})$$

$$= \int_{-2}^{2} (2 - (x^2 - 2))\, dx$$

$$= \int_{-2}^{2} (-x^2 + 4)\, dx$$

2) Find the area of the region enclosed by the line and the curve

$y = x^2 - 2$ and $y = 2$, above the x_axis



the area of (1) $= \int_c^d (\text{the right curve} - \text{left curve})\,dy$

$$= \int_0^2 (\sqrt{y+2} - 0)\,dy$$

So the area of the region $= 2\int_0^2 (\sqrt{y+2})\,dy$

3) Find the area of the region bounded by

$y = x^4 - 4x^2 + 4$ and $y = x^2$ over $[-1, 1]$

ans: First graph $y = x^4 - 4x^2 + 4$

$$\frac{dy}{dx} = 4x^3 - 8x = 4x(x^2 - 2) = 4x(x - \sqrt{2})(x + \sqrt{2})$$

$$\Rightarrow \frac{dy}{dx} = 0 \iff x = 0 \quad \text{or} \quad x = \sqrt{2} \quad \text{or} \quad x = -\sqrt{2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 12x^2 - 8 = 4(3x^2 - 2)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 0 \iff x = \pm\sqrt{\frac{2}{3}}$$

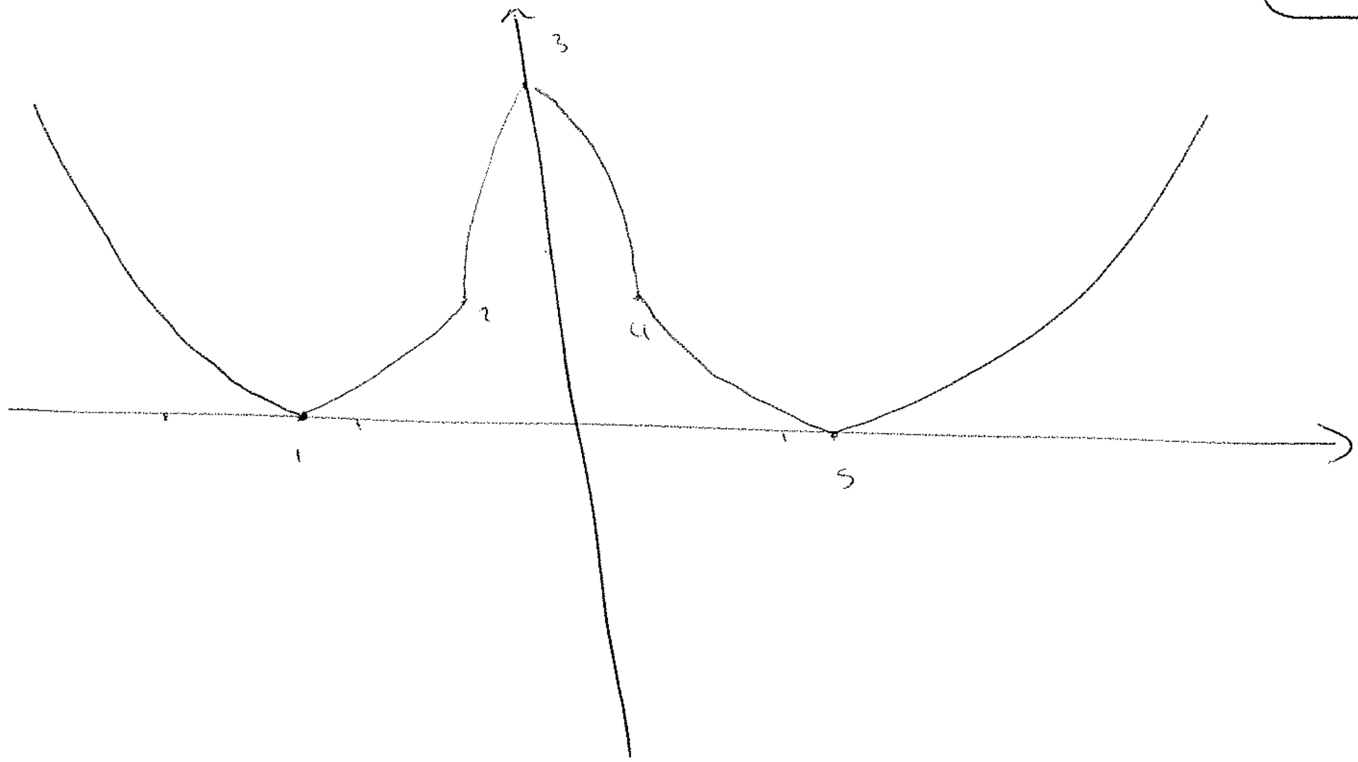
now, we have these points:

(3) $(0, 4)$, (5) $(\sqrt{2}, 0)$, (1) $(-\sqrt{2}, 0)$, (2) $\left(-\sqrt{\frac{2}{3}}, \frac{16}{9}\right)$,

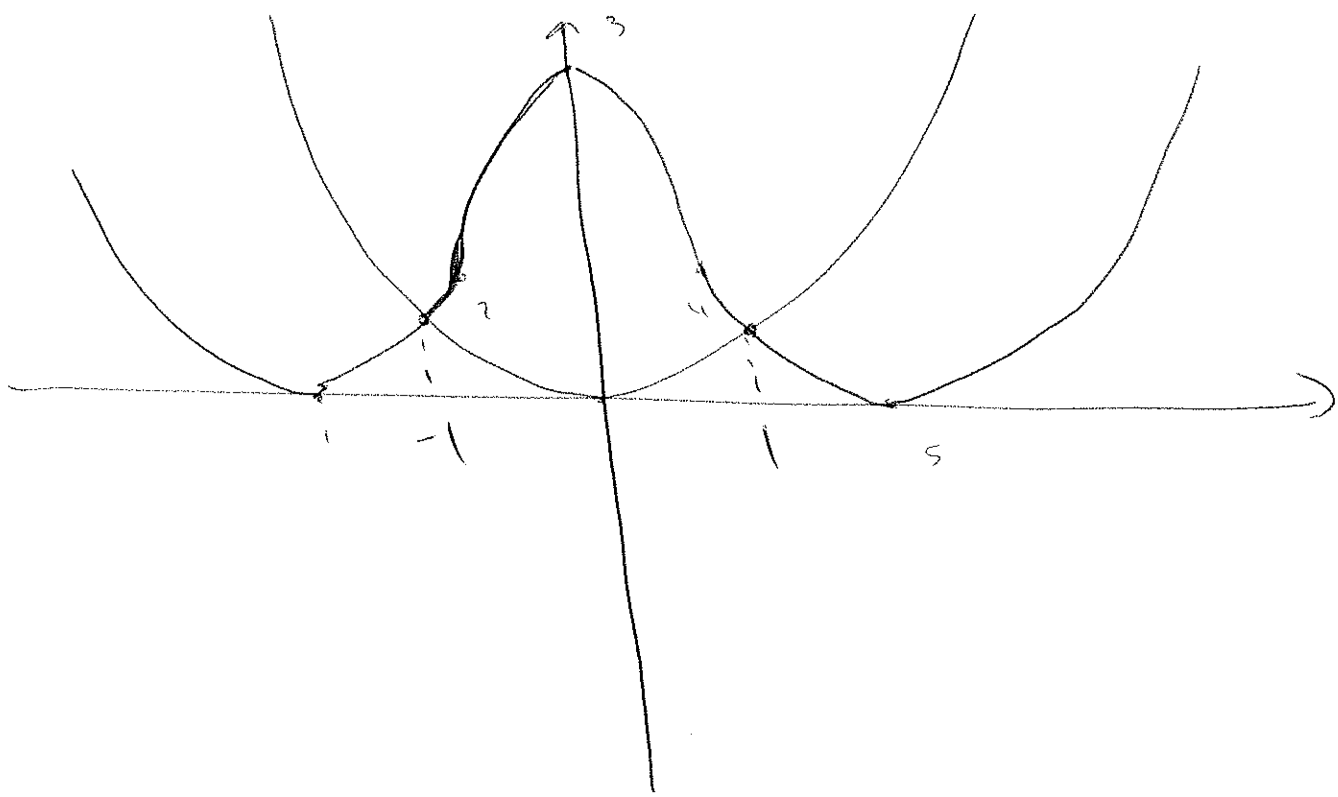
(4) $\left(\sqrt{\frac{2}{3}}, \frac{16}{9}\right)$

	$-\sqrt{2}$ $-\sqrt{\frac{2}{3}}$ 0 $\sqrt{2}$ $\sqrt{\frac{2}{3}}$
$4x$	− − − 0 + + +
$x - \sqrt{2}$	− − − − 0 + +
$x + \sqrt{2}$	− 0 + + + + +
$f'(x)$	− 0 + + 0 − 0 + +
$f''(x)$	+ + 0 − − − 0 +
$f(x)$	↘∪ ↗∪ ↗∩ ↘∩ ↗∪ ↗

so $y = x^4 - 4x^2 + 4$ is

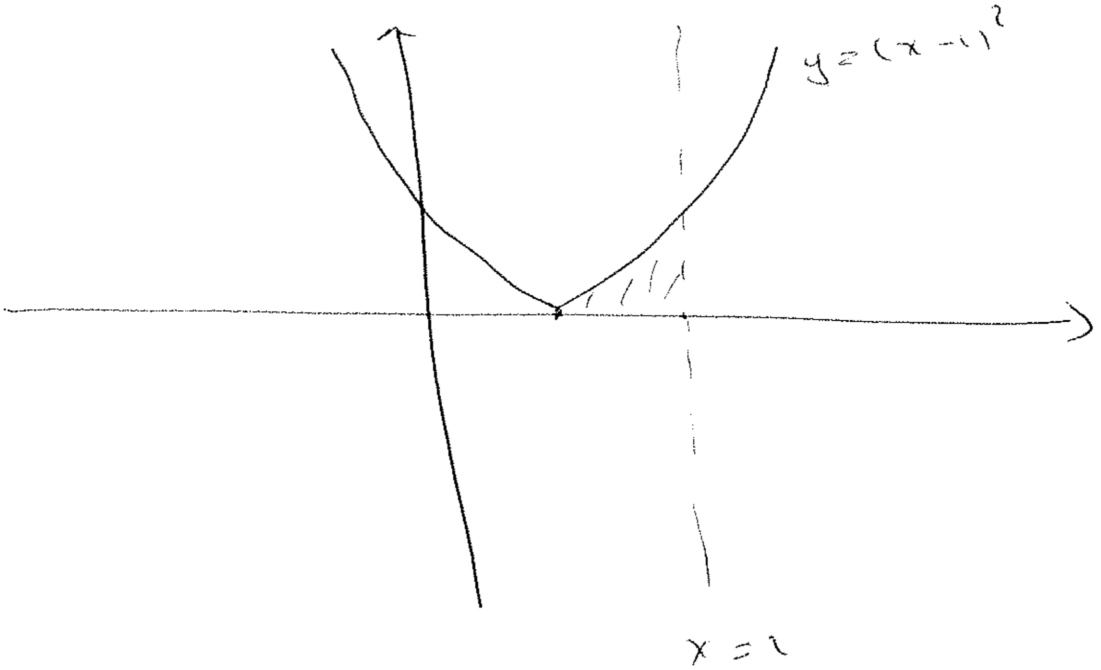


so the region is :



the area is $= \int_{-1}^{1} ((x^4 - 4x^2 + 4) - (x^2))\, dx$

5) Find the volume of the solid generated by revolving the regions bounded by the lines and curves: $y=(x-1)^2$, $y=0$, $x=2$



the volume $= \int_1^2 \pi (x-1)^4 \, dx$

If you know what the graph of f(x) is, then to find:

① $f(x-k) \Rightarrow$ shift right by k

② $f(x+k) \Rightarrow$ " left " "

③ $f(-x) \Rightarrow$ flip over the y-axis

6)

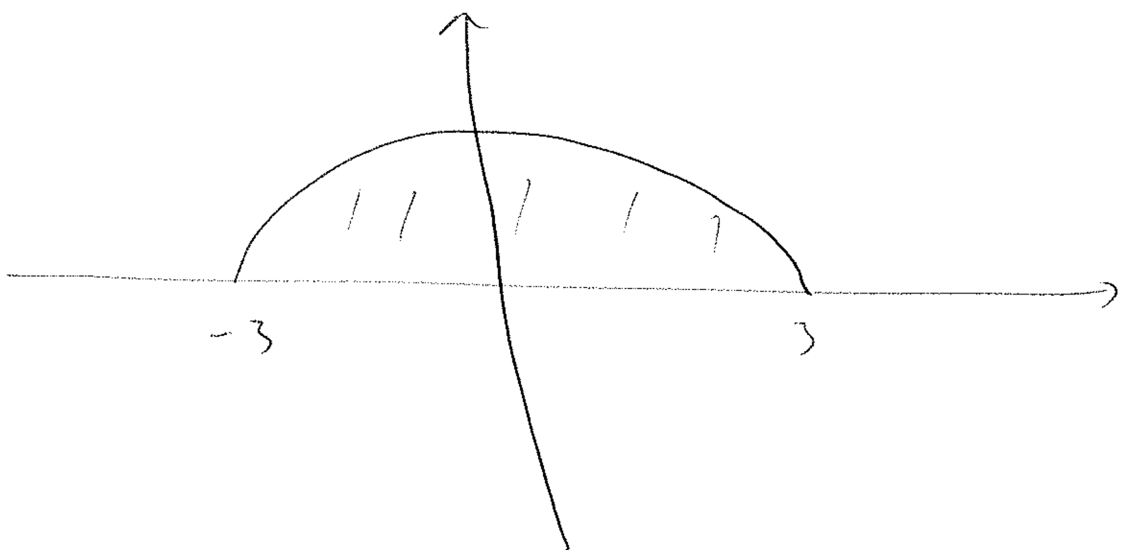
④ $f(x)+k \Rightarrow$ Shift up by k

⑤ $f(x)-k \Rightarrow$ Shift down by k

⑥ $-f(x) \Rightarrow$ flip over the x-axis

Find the volume of the solid generated by revolving the region bounded by the lines and curves:

$y = \sqrt{9-x^2}$, $y=0$



$$V = \int_{-3}^{3} \pi \left(\sqrt{9-x^2}\right)^2 dx$$