

# MA140-Engineering Calculus

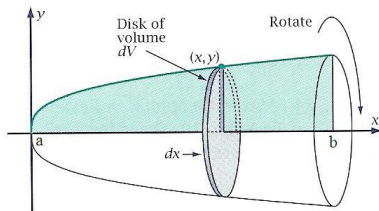
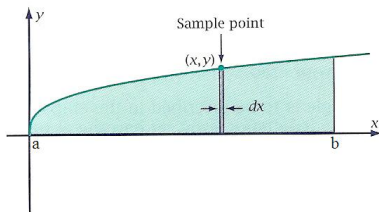
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Find the volume of solids:



The solid generated by rotating a plane region about an axis in its plane is called a solid of revolution. To find the volume of a solid we need only observe that the cross-sectional area  $A(x)$  is the area of a disk of radius  $R(x)$ , the distance of the planar regions boundary from the axis of revolution. The area is then

$$A(x) = \pi(\text{radius})^2 = \pi[R(x)]^2$$

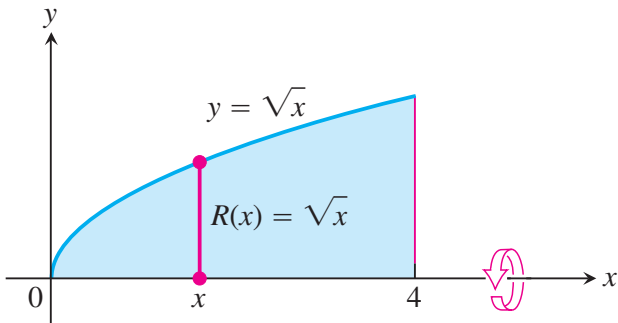
So the definition of volume gives:

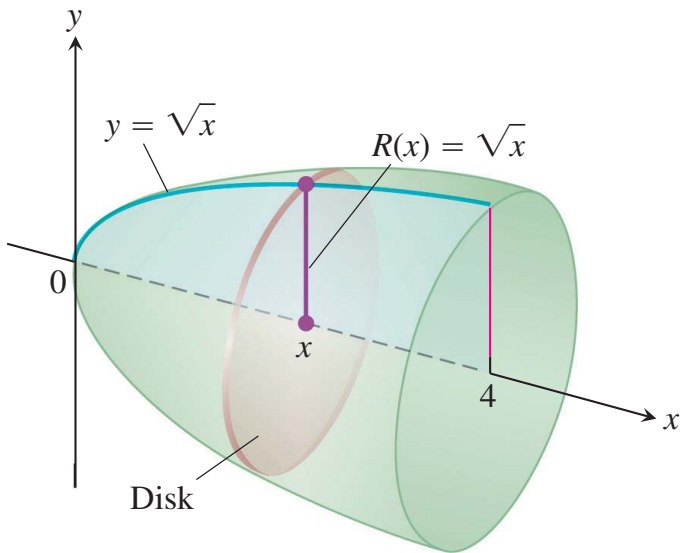
$$V = \int_a^b A(x)dx = \int_a^b \pi[R(x)]^2 dx$$

## Example 1.1

The region between the curve  $y = \sqrt{x}$ ,  $0 \leq x \leq 4$ , and the  $x$ -axis is revolved about the  $x$ -axis to generate a solid. Find its volume.

We draw figures showing the region, a typical radius, and the generated solid.





The volume is:

$$\begin{aligned} V &= \int_a^b \pi[R(x)]^2 dx == \int_0^4 \pi[\sqrt{x}]^2 dx \\ &= \pi \int_0^4 x dx = \pi \left. \frac{x^2}{2} \right]_0^4 = \pi \frac{(4)^2}{2} = 8\pi \end{aligned}$$

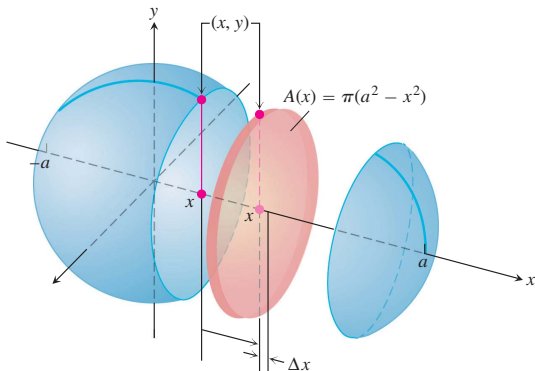
## Example 1.2

The circle

$$x^2 + y^2 = a^2$$

is rotated about the  $x$ -axis to generate a sphere. find its volume

We imagine the sphere cut into thin slices,



the cross-sectional area at a typical point  $x$  between  $-a$  and  $a$  is:

$$A(x) = \pi y^2 = \pi(a^2 - x^2)$$

Therefore, the volume is

$$\begin{aligned} V &= \int_{-a}^a A(x) dx = \int_{-a}^a \pi(a^2 - x^2) dx \\ &= \pi \left[ a^2 x - \frac{x^3}{3} \right]_{-a}^a = \frac{4}{3} \pi a^3 \end{aligned}$$

### Example 1.3

Sketch the area between  $x = 2$  and  $x = 3$ , under the curve

$$y = \frac{1}{x - 1}$$

Also find the volume of the solid if this area is rotated around the  $x$ -axis.