MA140-Engineering Calculus

Lecture 21

October 19, 2017

Example 1.1

Find $\frac{dy}{dx}$, if

$$y = \int_1^{x^2} \cos(t)dt$$

The upper limit of integration is not x but x^2. This makes y a composite of the two functions,

$$y = \int_1^{u} \cos(t)dt \quad and \quad u = x^2$$

We must therefore apply the chain rule when finding $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{d}{du}\int_1^{u} \cos(t)dt\right) \cdot \frac{du}{dx}$$

$$= \cos(u) \cdot \frac{du}{dx} = \cos(x^2) \cdot 2x = 2x \cdot \cos(x^2)$$

Example 1.2

Find $\frac{dy}{dx}$, if

$$y = \int_3^{\sqrt{x}} \frac{\cos(t)}{t} dt$$

By the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \left(\frac{d}{du} \int_3^u \frac{\cos(t)}{t} dt \right) \cdot \frac{du}{dx}$$

$$= \frac{\cos(u)}{u} \cdot \frac{du}{dx} = \frac{\cos(\sqrt{x})}{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

Integration by parts is a technique for simplifying integrals of the form:

$$\int f(x) \cdot g(x) dx$$

It is useful when f can be differentiated repeatedly and g can be integrated repeatedly without difficulty. The integral

$$\int xe^x dx$$

is such an integral because $f(x) = x$ can be differentiated twice to become zero and $g(x) = e^x$ can be integrated repeatedly without difficulty.

Theorem 1.3

Integration by Parts Formula:

$$\int udv = uv - \int vdu$$

Example 1.4

Find

$$\int x\cos(x)dx$$

We use the formula $\int udv = uv - \int vdu$ with

$$u = x, \quad dv = \cos(x)dx,$$

$$du = dx, \quad v = \sin(x)$$

Then

$$\int x\cos(x)dx = x\sin(x) - \int \sin(x)dx = x\sin(x) + \cos(x) + c$$

Example 1.5

Find

$$\int \ln(x)dx$$

Since

$$\int \ln(x)dx$$

can be written as

$$\int \ln(x) \cdot 1dx$$

, we use integration by parts:

$$u = \ln(x) \quad dv = dx$$

$$du = \frac{1}{x}dx, \quad v = x$$

Then:

$$\int \ln(x)dx = x\ln(x) - \int dx = x\ln(x) - x + c$$