# MA140-Engineering Calculus

Lecture 2

September 7, 2017

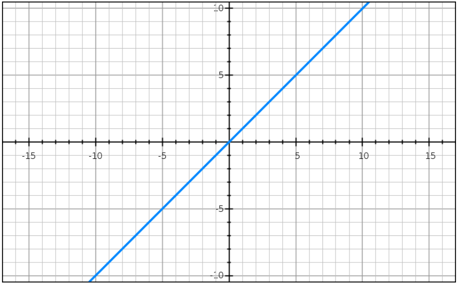
A polynomial is a function of the form:

$$y = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n, a_{n-1}, \cdots, a_0$ are constants.
These constants are called the *coefficients* of the polynomial.
The number $n$ is the degree of the polynomial.

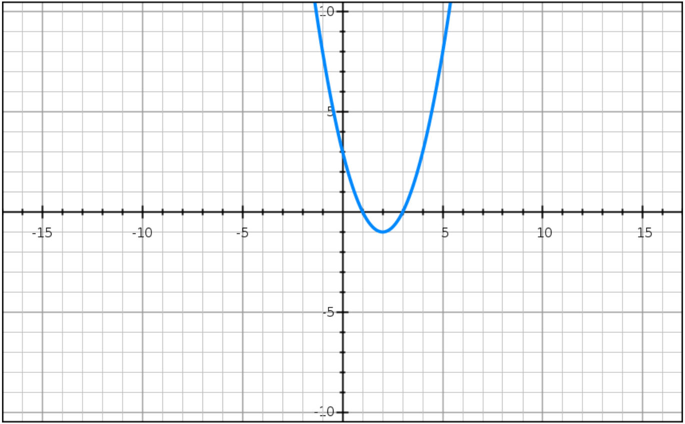
## Example 1.1

$y = x$, is a *linear* polynomial



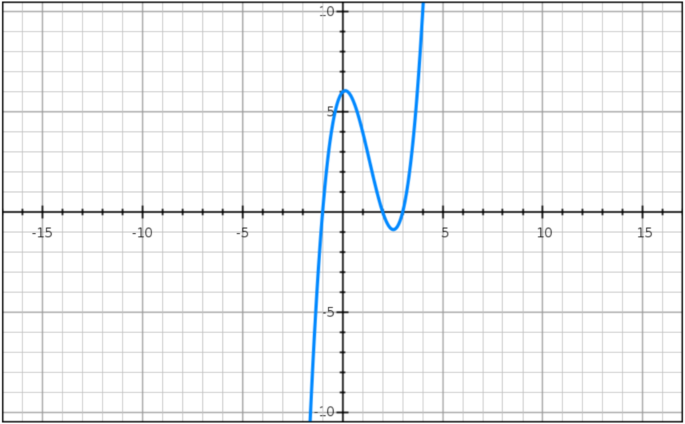
## Example 1.2

$y = x^2 - 4x + 3$ is a *quadratic* polynomial.



## Example 1.3

$y = x^3 - 4x^2 + x + 6$ is a *cubic* polynomial.

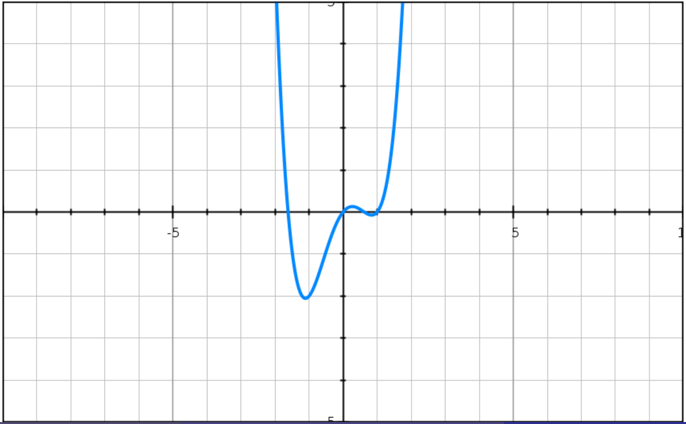


## General facts on polynomial sketching:

A polynomial of degree $n$ has up to $n-1$ bends.

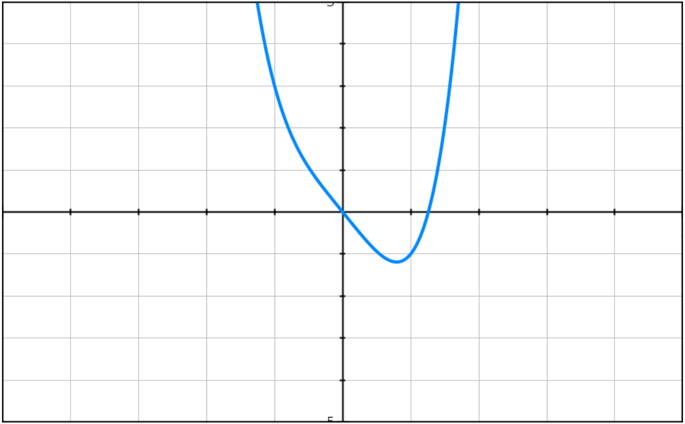
### Example 1.4

a typical fourth degree polynomial has $3$ bends. for example
$y = x^4 - 2x^2 + x$



## Example 1.5

the fourth degree polynomial $y = x^4 - 2x$ has only one bend.

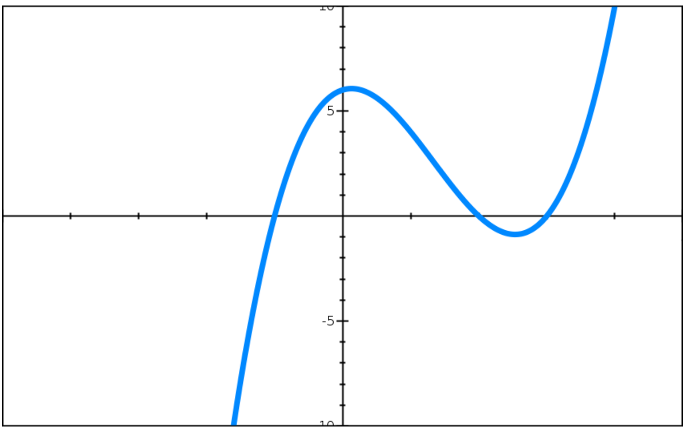


**Find the intercepts:**
The $y-$intercept can be found by letting $x = 0$.
The $x-$intercepts are the roots (or zeros).
**Note:** you do not have to use the same scale on the $x$ and $y$ axis.

**Example 1.6**

$y = x^3 - 4x^2 + x + 6$

$x = 0 \;\; \Rightarrow \;\; y = 6$ ($y-$intercept)
(The constant coefficient$=-$ product of the roots if the coefficient of the highest power$=1$)
By trial roots are $x = -1, 2, 3$


5
-5

## Definition 1.7

Rational functions have the general form

$$f(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomials.

- **IF** degree of $p(x) <$ degree of $q(x)$, then $f(x)$ is a strictly proper rational function.
- **IF** degree of $p(x) =$ degree of $q(x)$, then $f(x)$ is a proper rational function.
- **IF** degree of $p(x) >$ degree of $q(x)$, then $f(x)$ is an improper rational function.

An improper or proper rational function can be expressed in terms of a strictly proper rational function

### Example 1.8

Express $f(x) = \dfrac{3x^4 + 2x^3 - 5x^2 + 6x - 7}{x^2 - 2x + 3}$ in terms of a strictly proper rational function

$$f(x) = 3x^2 + 8x + 2 - \frac{14x + 13}{x^2 - 2x + 3}$$