MA140-Engineering Calculus

Lecture 18

October 15, 2017

Definition 1.1

A function F is an *antiderivative* of f on an interval I if $F'(x) = f(x)$ for all x in I.
f is the derivative of $F \longleftrightarrow F$ is an antiderivative of f

Note: If F is an antiderivative of f, then the most general antiderivative of f is

$$F(x) + c$$

Where c is an arbitrary constant.
For example the antiderivative of $f(x) = 2x$ equals $F(x) = x^2 + c$
or
For example the antiderivative of $f(x) = 3x^2$ equals $F(x) = x^3 + c$

Definition 1.2

We call

$$\int f(x)dx$$

an *indefinite integral* if

$$\int f(x)dx = F(x) + c$$

where $F(x)$ is an antiderivative of $f(x)$.

Example 1.3

- $\int 2xdx = x^2 + c$
- $\int 3x^2dx = x^3 + c$

Example 1.4

- $$\int x dx = \frac{1}{2}x^2 + c$$

  Because the derivative of $\frac{1}{2}x^2 + c$ is equal to x

- $$\int x^2 dx = \frac{1}{3}x^3 + c$$

In general when $x \neq -1$:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Example 1.5

- $$\int \sin(x)dx = -\cos(x) + c$$
- $$\int \cos(x)dx = \sin(x) + c$$
- $$\int \sec^2(x)dx = \tan(x) + c$$
- $$\int \csc^2(x)dx = -\cot(x) + c$$
- $$\int \sec(x)\tan(x)dx = \sec(x) + c$$
- $$\int \csc(x)\cot(x)dx = -\csc(x) + c$$

Some useful integral properties:

(1)

$$\int cf(x)dx = c\int f(x)dx$$

(2)

$$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$$

Example 1.6

evaluate the following integral:

$$\int (2x^2 + 9x^7)dx$$

$$\int (2x^2 + 9x^7)dx = \int 2x^2dx + \int 9x^7dx = 2\int x^2dx + 9\int x^7dx$$

$$= [2(\frac{x^3}{3}) + c_1] + [9(\frac{x^8}{8}) + c_2] = 2(\frac{x^3}{3}) + 9(\frac{x^8}{8}) + (c_1 + c_2)$$

$$\int (2x^2 + 9x^7)dx = 2(\frac{x^3}{3}) + 9(\frac{x^8}{8}) + c$$

Theorem 1.7

If $u = g(x)$ is a differentiable function, then:

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Example 1.8

evaluate the following integral:

$$\int 3x^2 \sin(x^3)dx$$

We see that $3x^2$ is the derivative of x^3. So if we make the substitution $u = x^3$, then $\frac{du}{dx} = 3x^2$ or in the differential form $du = 3x^2dx$, so

$$\int 3x^2 \sin(x^3)dx = \int \sin(u)du$$

Now we integrate with respect to u:

$$\int \sin(u)du = -\cos(u) + c$$

Replace u by x^3, we get:

$$\int 3x^2 \sin(x^3)dx = \int \sin(u)du = \int \sin(u)du = -\cos(u)+c = -\cos(x^3)+c$$

Example 1.9

Find:

$$\int 2x\sqrt{1+x^2}dx$$

We define a function of x, called u.

Let $u = 1 + x^2$, so $\frac{du}{dx} = 2x$ or $du = 2xdx$ So the above integral becomes:

$$\int \sqrt{u}du = \int u^{\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}u^{\frac{3}{2}} + c = \frac{2}{3}(1+x^2)^{\frac{3}{2}} + c$$

Example 1.10

Find:

$$\int \cos(4x-7)dx$$

Let $u = 4x - 7$. then $\dfrac{du}{dx} = 4$ or $du = 4dx$.
We can rewrite the original integral as:

$$\frac{1}{4}\int 4\cos(4x-7)dx$$

then the above integral equals:

$$\frac{1}{4}\int \cos(u)du = \frac{1}{4}\sin(u) + c = \frac{1}{4}\sin(4x-7) + c$$

Example 1.11

Find:

$$\int e^{-5x} dx$$

Let $u = -5x$, then $\frac{du}{dx} = -5$ or $du = -5dx$.
We can rewrite the original integral as:

$$\frac{-1}{5} \int -5e^{-5x} dx$$

then the above integral equals:

$$\frac{-1}{5} \int e^{u} du = \frac{-1}{5} e^{u} + c = \frac{-1}{5} e^{-5x} + c$$

Example 1.12

Find:

$$\int \sin^3 x \cos x dx$$