MA140-Engineering Calculus

Lecture 16

October 15, 2017

Logarithmic Differentiation

One could use the product and quotient rule to differentiate the following type of function.

Example 1.1

$$f(x) = \frac{(x^2+5)(x+3)^2\sqrt{x+7}}{(x+4)^5}$$

Alternatively, we could first take the log of both sides:

$$\ln f(x) = \ln(x^2+5) + 2\ln(x+3) + \frac{1}{2}\ln(x+7) - 5\ln(x+4)$$

then differentiate
First note:

$$\frac{d}{dx}(\ln f(x)) = \frac{d\ln f(x)}{df(x)} \cdot \frac{df(x)}{dx} = \frac{1}{f(x)} \cdot \frac{df(x)}{dx} = \frac{1}{f(x)} \cdot f'(x)$$

So differentiating the above:

$$\frac{f'(x)}{f(x)} = \frac{2x}{x^2+5} + \frac{2}{x+3} + \frac{1}{2(x+7)} - \frac{5}{x+4}$$

So differentiating the above:

$$\frac{f'(x)}{f(x)} = \frac{2x}{x^2+5} + \frac{2}{x+3} + \frac{1}{2(x+7)} - \frac{5}{x+4}$$

So

$$f'(x) = \left[\frac{(x^2+5)(x+3)^2\sqrt{x+7}}{(x+4)^5}\right] \cdot \left[\frac{2x}{x^2+5} + \frac{2}{x+3} + \frac{1}{2(x+7)} - \frac{5}{x+4}\right]$$

Continuous and Differentiable Function

Example 1.2

Determine values of k and m such that $h(x)$ is continuous and differentiable at all points.

$$h(x) = \begin{cases} 4x + k & if \quad x < 3 \\ mx^3 - 1 & if \quad x \geq 3 \end{cases}$$

To be continuous:

$$\lim_{x \to 3^-} h(x) = \lim_{x \to 3^+} h(x)$$

So,

$$4(3) + k = m(3)^3 - 1 \Rightarrow 12 + k = 27m - 1 \Rightarrow k = 27m - 13$$

A function $h(x)$ is differentiable at a if the following limit exists:

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

this limit exists, if both of the one-sided limits exist and are equal:

$$\lim_{x \to 3^-} \frac{(4x + k) - (27m - 1)}{x - 3} = \lim_{x \to 3^-} \frac{4x - 12}{x - 3} = 4$$

Also

$$\lim_{x \to 3^+} \frac{(mx^3 - 1) - (27m - 1)}{x - 3} = \lim_{x \to 3^+} \frac{m(x^3 - 27)}{x - 3}$$

$$= \lim_{x \to 3^+} m(x^2 + 3x + 9) = 27m$$

so

$$4 = 3m(3^2) \Rightarrow 4 = 27m \Rightarrow m = 27/4$$

As $k = 27m - 13$, $k = 27(4/27) - 13 \Rightarrow k = 4 - 13 = -9$