MA140-Engineering Calculus

Lecture 15

October 6, 2017

Example 1.1

Sketch the graph of the function

$$f(x) = \frac{1}{1 + e^{3x}}$$

Step (1):
Find the critical points, we can write $f(x)$ as, $f(x) = (1 + e^{3x})^{-1}$, so

$$f'(x) = -3(e^{3x})(1 + e^{3x})^{-2} = \frac{-3e^{3x}}{(1 + e^{3x})^2}$$

As for all $x \in \mathbb{R}$, $e^{3x} > 0$, so $f'(x)$ has no root and it is always negative which means that the function is always decreasing.
Step (2):
Find the points of inflection:

$$f''(x) = \frac{(1 + e^{3x})^2[-9e^{3x}] - [-3e^{3x}][6e^{3x}(1 + e^{3x})]}{(1 + e^{3x})^4}$$

Setting $f''(x) = 0$, we have

$$(1 + e^{3x})^2(-9e^{3x}) + [3e^{3x}][6e^{3x}(1 + e^{3x})] = 0$$

$$\Rightarrow (1 + e^{3x})(-9) + 18e^{3x} = 0 \Rightarrow 1 + e^{3x} = 2e^{3x} \Rightarrow 1 = e^{3x}$$

So $x = 0$ could be the point of inflection, we need to check whether the second derivative changes sign around this point or not.

Step (3):

No need to use the second derivative test, as there is no critical point.

Step (4):

$f(0) = 1/2$, the function has no x-intercept as $f(x)$ is never zero.

As $f(x)$ is a rational function. we should also find the asymptotes

$$\lim_{x\to\infty} \frac{1}{1+e^{3x}} = 0$$

also

$$\lim_{x\to-\infty} \frac{1}{1+e^{3x}} = \frac{1}{1+0} = 1$$

Is the denominator equal to zero? No, because there is no real x when $1+e^{3x}=0$

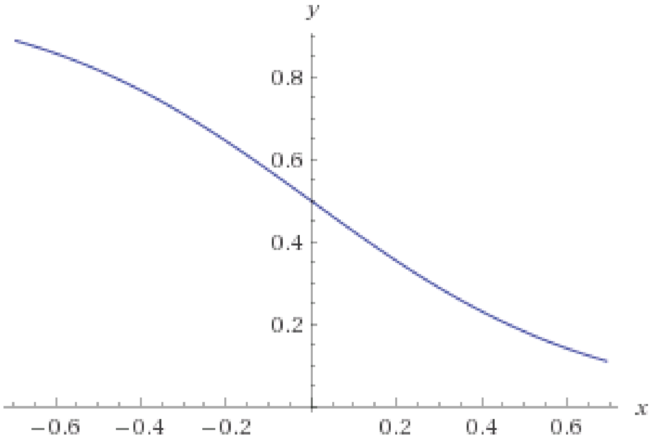
So the asymptotes are:

$y=0$ when $x\to\infty$

$y=1$ when $x\to-\infty$

Step 5 and 6: $f'(x) = \dfrac{-3e^{3x}}{(1+e^{3x})^2}$ and $f''(x) = \dfrac{9(e^{3x}-1)}{(1+e^{3x})^4}$

	0	
$f'(x)$	$-$	$-$
$f''(x)$	$-$	$+$



y
0.8
0.6
0.4
0.2
x
−0.6
−0.4
−0.2
0.2
0.4
0.6