

MA140-Engineering Calculus

Lecture 14

October 6, 2017

Example 1.1

sketch a graph of the function $f(x) = x^4 - 4x^3 + 10$

We use the following steps:

- (1) Find the critical (stationary) points
- (2) Find the points of inflection
- (3) Use the second derivative test
- (4) Find the y-value of these points to pair them
- (5) Draw the table to find the intervals on which f is increasing and the intervals on which f is decreasing
- (6) Add some extra rows to your table to see where the graph of f is concave up and where it is concave down
- (7) Plot some specific points, such as local maximum and minimum points, points of inflection, and intercepts.
- (8) Sketch the general shape of the graph for f

Step (1):

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) = 0$$

So the stationary points are $x = 0$ and $x = 3$

Step (2):

$$f''(x) = 12x^2 - 24x = 12x(x - 2) = 0$$

So the points of inflection are $x = 0$ and $x = 2$

Step (3):

At $x = 0$, $f''(0) = 0$ so the test fails in this point. But at $x = 3$, $f''(3) = 36 > 0$ so based on the test, $x = 3$ is a local minimum

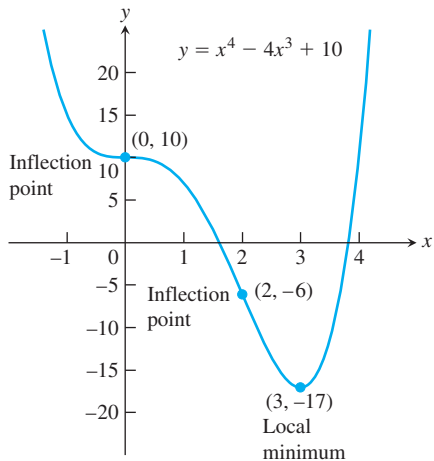
Step 4:

$$f(0) = 10, f(2) = -6 \text{ and } f(3) = -17$$

Step 5 and 6:

	0		2		3	
$4x^2$	+	•	+		+	
$x - 3$	-		-		-	•
$f'(x)$	-	•	-		-	•
$12x$	-	•	+		+	
$x - 2$	-		-	•	+	
$f''(x)$	+	•	-	•	+	

Step 7 and 8:



Example 1.2

Sketch the graph of

$$f(x) = \frac{(x+1)^2}{1+x^2}$$

Step (1):

$$f(x) = \frac{(x+1)^2}{1+x^2}, \text{ so}$$

$$f'(x) = \frac{(1+x^2) \cdot 2(x+1) - (x+1)^2 \cdot 2x}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)^2}$$

So the critical points are $x = -1$ and $x = 1$.

Step (2):

$$f''(x) = \frac{(1+x^2)^2 \cdot 2(-2x) - 2(1-x^2)[2(1+x^2) \cdot 2x]}{(1+x^2)^4} = \frac{4x(x^2-3)}{(1+x^2)^3}$$

So inflection points are $x = \sqrt{3}$, $x = 0$ and $x = -\sqrt{3}$

Step (3):

since $f''(1) = -1 < 0$ so $x = 1$ is a local max and since $f''(-1) = 1 > 0$ so $x = -1$ is a local min

Step (4):

$$f(1) = 2, f(-1) = 0, f(\sqrt{3}) = \frac{(\sqrt{3}+1)^2}{4}, f(-\sqrt{3}) = \frac{(-\sqrt{3}+1)^2}{4}, f(0) = 1$$

We can also find the asymptotes of the function:

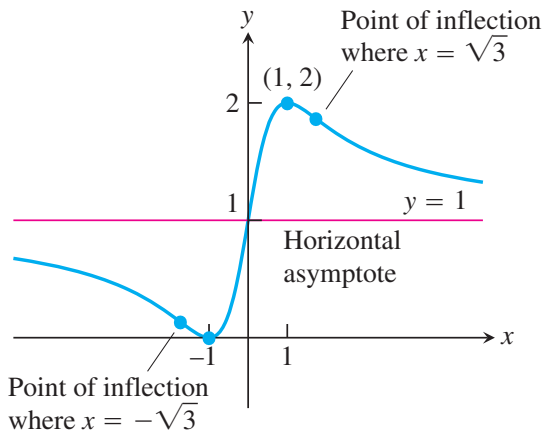
$$\lim_{x \rightarrow \infty} f(x) = 1$$

So $y = 1$ is the horizontal asymptote.

Step 5 and 6:

	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$		
$2(1-x^2)$	-	-	•	+	+	•	-
$(1+x^2)^2$	+	+	+	+	+	+	+
$f'(x)$	-	-	•	+	+	•	-
$4x$	-	-	-	•	+	+	+
(x^2-3)	+	•	-	-	-	-	•
$(1+x^2)^3$	+	+	+	+	+	+	+
$f''(x)$	-	•	+	+	•	-	-

Step 7 and 8:



Exercise

Use the steps of the graphing procedure to graph the following equations:



$$y = x^2 - 4x + 3$$



$$y = x^3 - 3x + 3$$



$$y = \frac{x^2 - 3}{x - 2}$$