

# MA140-Engineering Calculus

## Lecture 13

October 6, 2017

## Definition 1.1

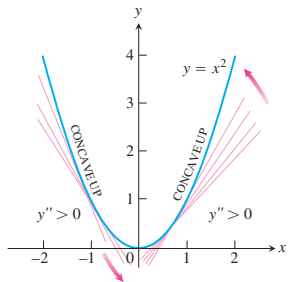
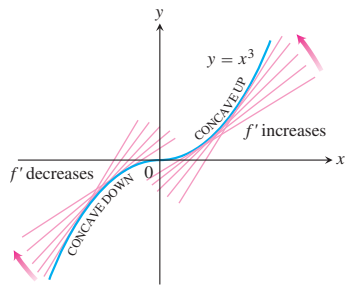
The graph of a differentiable function  $y = f(x)$  is:

- **concave up** on an open interval  $I$  if  $f'$  is increasing on  $I$
- **concave down** on an open interval  $I$  if  $f'$  is decreasing on  $I$

## Theorem 1.2

Let  $y = f(x)$  be twice-differentiable on an open interval  $I$ .

- If  $f'' > 0$  on  $I$ , the graph of  $f$  over  $I$  is concave up
- If  $f'' < 0$  on  $I$ , the graph of  $f$  over  $I$  is concave down.



### Definition 1.3

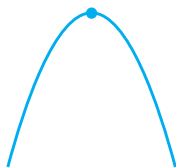
A point where the graph of a function has a tangent line and where the concavity changes is a **point of inflection**

*Note:* To find the points of inflection, we need to find the zeros of  $f''$  and points where  $f''$  is not defined. Then we need to check the concavity.

### Example 1.4

The curve  $y = x^4$  has no inflection point at  $x = 0$ . Even though  $y'' = 12x^2$  is zero there, it does not change sign

*Note:* An inflection point may not exist where  $y'' = 0$



$$f' = 0, f'' < 0 \\ \Rightarrow \text{local max}$$



$$f' = 0, f'' > 0 \\ \Rightarrow \text{local min}$$

## Theorem 1.5

**Second derivative test for local maximums and minimums:** suppose that  $f''$  is continuous on an open interval that contains  $x = c$ .

- If  $f'(c) = 0$  and  $f''(c) < 0$ , then  $f$  has a local maximum at  $x = c$
- If  $f'(c) = 0$  and  $f''(c) > 0$ , then  $f$  has a local minimum at  $x = c$
- If  $f'(c) = 0$  and  $f''(c) = 0$ , then the test fails. The function  $f$  may have a local maximum, a local minimum, or neither.

## Example 1.6

Find and classify the stationary points and the points of inflection of

$$f(x) = 4x^3 - 21x^2 + 18x + 6$$

$$f'(x) = 12x^2 - 42x + 18$$

When  $f'(x) = 0$ , we have:

$$12x^2 - 42x + 18 = 0 \Rightarrow 2x^2 - 7x + 3 = 0 \Rightarrow (2x - 1)(x - 3) = 0$$

So The stationary points are:  $x = 1/2$  and  $x = 3$

$$f''(x) = 24x - 42 \text{ so}$$

$$f''(1/2) = 24(1/2) - 42 = 12 - 42 < 0$$

which means  $x = 1/2$  is a local max. Also as

$$f''(3) = 24(3) - 42 = 72 - 42 > 0$$

$x = 3$  is a local min.