MA140-Engineering Calculus

Lecture 1

October 7, 2019

- **Lectures:** 10 am, ENG-G018, Tuesday, Wednesday, Thursday
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- **Tutorials:** details will be announced later, normally they start two weeks after the first lecture
- **Supporting centre:** SUMS, visit the homepage to see the timetables: http://www.maths.nuigalway.ie/sums/
- **Text books:** Modern Engineering Mathematics by Glyn James, Thomas' Calculus or any basic calculus text book

Lecture 1

In this lecture we review real and complex numbers.

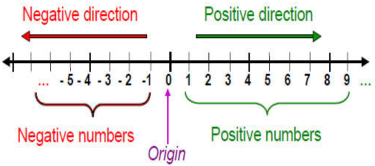
- Natural Numbers=$\mathbb{N} = \{1, 2, 3, \cdots\}$
  $1 + 3 = 4$, 4 is a natural number or 4 belongs to the set of natural numbers $\simeq 4 \in \mathbb{N}$

$$1 - 1 = 0, \quad 0 \notin \mathbb{N}$$

- Whole Numbers=$\mathbb{N}_0 = \{0, 1, 2, \cdots\}$ so we see that $2 - 2 = 0 \in \mathbb{N}_0$ but $1 - 3 = -2 \notin \mathbb{N}_0$
  $\mathbb{N}$ is a subset of $\mathbb{N}_0 \quad \longleftrightarrow \quad \mathbb{N} \subset \mathbb{N}_0$
  Remark:
  - $A \subset B$ it means for all elements in A or for any element in A like x then x is in $B \quad \longleftrightarrow \quad (\forall x \in A \quad \to x \in B)$
    So $\forall$ means "for all"
  - $A \not\subset B$ it means there exists an element in A like x which is not in $B \quad \longleftrightarrow \quad (\exists x \in A \quad .s.t \quad x \notin B)$

- Integers=$\mathbb{Z} = \{\cdots, -2, -1, 0, 1, 2, \cdots\}$
  $1 + (-4) = -3 \in \mathbb{Z}$ and $1 - 5 = -4 \in \mathbb{Z}$ so the addition and subtraction of two integers is again an integer but what about the division?
  $\frac{4}{2} = 2 \in \mathbb{Z}$, $(\frac{4}{3}) \notin \mathbb{Z}$
- Rational Numbers=$\mathbb{Q} = \{\frac{a}{b} \quad | \quad a, b \in \mathbb{Z} \quad and \quad b \neq 0\}$
  for example $\frac{-3}{4} = -0.75000...$ or $\frac{1}{3} = -0.33333...$
  But $\sqrt{2} \notin \mathbb{Q}$

- Real Numbers=$\mathbb{R}$, the set of real numbers includes all the rational numbers and numbers like $\sqrt{2}, \pi = 3.14..., 0.11236$, these numbers are called irrational numbers so
  real numbers=(rational numbers)∪(irrational numbers)
  $\mathbb{R} = \mathbb{Q} \cup (\mathbb{R} \backslash \mathbb{Q})$



$$\mathbb{N} \subset \mathbb{N}_0 \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

- Complex Numbers=$\mathbb{C}$
  If $c \in \mathbb{C}$, we can write $c = a + ib, \quad a, b \in \mathbb{R} \quad and \quad i = \sqrt{-1}$

$$\mathbb{N} \subset \mathbb{N}_0 \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

We represent a function in one of the two ways:

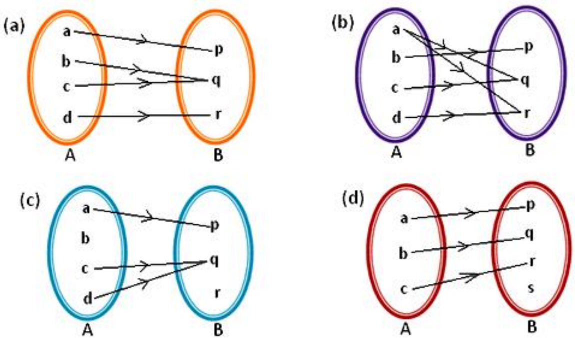
$$f : x \to y \quad or \quad y = f(x)$$

- x is called the independent variable and y is called the dependent variable.
- when we write $y = f(x)$, "x" is known as the *argument* of the function.
- here x is in the set X and the set X is called the *domain* of the function
- and y is in the set Y, the set Y is called the *codomain*

Definition 1.1

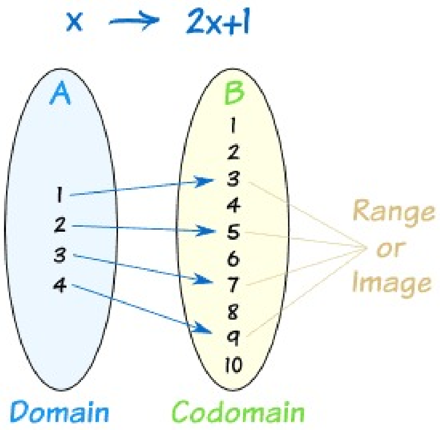
A *function* from a set X to a set Y is a rule that assigns a unique (single) element $f(x) \in Y$ to each element $x \in X$.

Note: f associates each value of x in X with exactly one value of y in Y. It means we can not have different outputs for the same input.



- when $y = f(x)$, y is said to be the *image* of x under f
- the set of all images is called the *image set* or *range* of f

Note: It is not necessary for all elements of the codomain set Y to be images under f



Example 1.2

Identify the domain, codomain and range of

(a) $f(x) = 3x^2 + 1$

(b) $f(x) = \sqrt{(x+4)(3-x)}$

solution (a):

$f(x)$ can be evaluated for all $x \in \mathbb{R}$, so domain$=\mathbb{R}$.

The lowest value $f(x)$ can take is 1 (when $x = 0$) so range$=[1, \infty]$.

We could write this as $\{y \mid y \geq 1 \;,\; y \in \mathbb{R}\}$.

We could take the codomain as $\mathbb{R}$ as it contains the range.

solution (b):

The domain is $[-4, 3]$, outside this range the function is not real valued i.e. it involves $\sqrt{-1}$.

The function takes value 0 at $x = -4$ and $x = 3$ and takes $7/2$, its highest value at $x = -1/2$. Therefore its range is $[0, 7/2]$. we can take the codomain to be $\mathbb{R}$ (as it contains the range)