

# MA140-Engineering Calculus

## Lecture 1

September 6, 2017

- **Lectures:** 10 am, ENG-G018, Tuesday, Wednesday, Thursday
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- **Tutorials:** details will be announced later, normally they start two weeks after the first lecture
- **Supporting centre:** SUMS, visit the homepage to see the timetables: <http://www.maths.nuigalway.ie/sums/>
- **Text books:** Modern Engineering Mathematics by Glyn James, Thomas' Calculus or any basic calculus text book

# Lecture 1

In this lecture we review real and complex numbers.

- Natural Numbers= $\mathbb{N} = \{1, 2, 3, \dots\}$   
 $1 + 3 = 4$ , 4 is a natural number or 4 belongs to the set of natural numbers  $\simeq 4 \in \mathbb{N}$

$$1 - 1 = 0, \quad 0 \notin \mathbb{N}$$

- Whole Numbers= $\mathbb{N}_0 = \{0, 1, 2, \dots\}$  so we see that  $2 - 2 = 0 \in \mathbb{N}_0$   
 but  $1 - 3 = -2 \notin \mathbb{N}_0$   
 $\mathbb{N}$  is a subset of  $\mathbb{N}_0 \iff \mathbb{N} \subset \mathbb{N}_0$

Remark:

- $A \subset B$  it means for all elements in  $A$  or for any element in  $A$  like  $x$  then  $x$  is in  $B \iff (\forall x \in A \rightarrow x \in B)$   
 So  $\forall$  means "for all"
- $A \not\subset B$  it means there exists an element in  $A$  like  $x$  which is not in  $B \iff (\exists x \in A .s.t \ x \notin B)$

- Integers= $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$   
 $1 + (-4) = -3 \in \mathbb{Z}$  and  $1 - 5 = -4 \in \mathbb{Z}$  so the addition and subtraction of two integers is again an integer but what about the division?

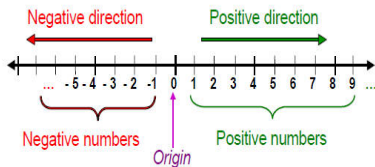
$$\frac{4}{2} = 2 \in \mathbb{Z}, \left(\frac{4}{3}\right) \notin \mathbb{Z}$$

- Rational Numbers= $\mathbb{Q} = \left\{\frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}$

for example  $\frac{-3}{4} = -0.75000\dots$  or  $\frac{1}{3} = 0.33333\dots$

But  $\sqrt{2} \notin \mathbb{Q}$

- Real Numbers= $\mathbb{R}$ , the set of real numbers includes all the rational numbers and numbers like  $\sqrt{2}$ ,  $\pi = 3.14\dots$ ,  $0.11236$ , these numbers are called irrational numbers so  
 real numbers=(rational numbers) $\cup$ (irrational numbers)  
 $\mathbb{R} = \mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$



$$\mathbb{N} \subset \mathbb{N}_0 \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

- Complex Numbers= $\mathbb{C}$

If  $c \in \mathbb{C}$ , we can write  $c = a + ib$ ,  $a, b \in \mathbb{R}$  and  $i = \sqrt{-1}$

$$\mathbb{N} \subset \mathbb{N}_0 \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

We represent a function in one of the two ways:

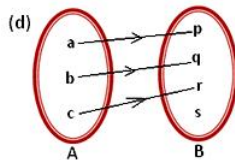
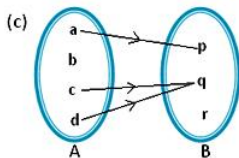
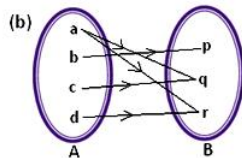
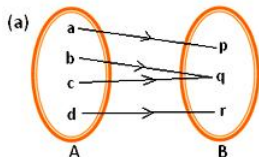
$$f : x \rightarrow y \quad \text{or} \quad y = f(x)$$

- $x$  is called the independent variable and  $y$  is called the dependent variable.
- when we write  $y = f(x)$ , " $x$ " is known as the *argument* of the function.
- here  $x$  is in the set  $X$  and the set  $X$  is called the *domain* of the function
- and  $y$  is in the set  $Y$ , the set  $Y$  is called the *codomain*

### Definition 1.1

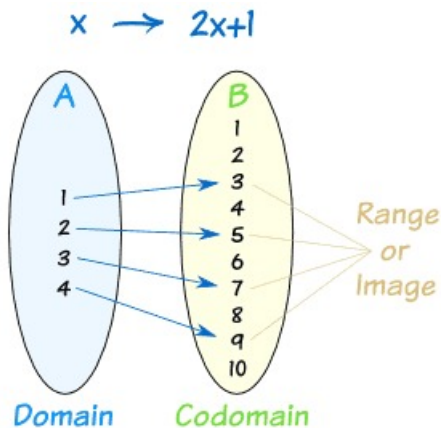
A *function* from a set  $X$  to a set  $Y$  is a rule that assigns a **unique** (single) element  $f(x) \in Y$  to each element  $x \in X$ .

**Note:**  $f$  associates each value of  $x$  in  $X$  with exactly one value of  $y$  in  $Y$ . It means we can not have different outputs for the same input.



- when  $y = f(x)$ ,  $y$  is said to be the *image* of  $x$  under  $f$
- the set of all images is called the *image set* or *range* of  $f$

**Note:** It is not necessary for all elements of the codomain set  $Y$  to be images under  $f$



## Example 1.2

Identify the domain, codomain and range of

(a)  $f(x) = 3x^2 + 1$

(b)  $f(x) = \sqrt{(x+4)(3-x)}$

**solution (a):**

$f(x)$  can be evaluated for all  $x \in \mathbb{R}$ , so domain= $\mathbb{R}$ .

The lowest value  $f(x)$  can take is 1 (when  $x = 0$ ) so range= $[1, \infty]$ .

We could write this as  $\{y \mid y \geq 1, y \in \mathbb{R}\}$ .

We could take the codomain as  $\mathbb{R}$  as it contains the range.

**solution (b):**

The domain is  $[-4, 3]$ , outside this range the function is not real valued i.e. it involves  $\sqrt{-1}$ .

The function takes value 0 at  $x = -4$  and  $x = 3$  and takes  $7/2$ , its highest value at  $x = -1/2$ . Therefore its range is  $[0, 7/2]$ . we can take the codomain to be  $\mathbb{R}$  (as it contains the range)