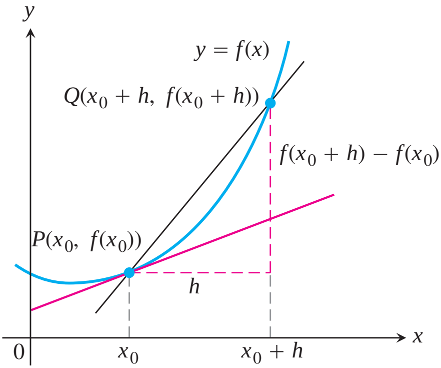
MA140-Engineering Calculus

Lecture 9

September 22, 2017

The change in distance $= f(x_0 + h) - f(x_0)$
The change in time $= (x_0 + h) - x_0 = h$
The average speed between P and Q is

$$\frac{f(x_0 + h) - f(x_0)}{h}$$



The slope of the curve $y = f(x)$ at the point $P(x_0, f(x_0))$ is the number

$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

We call this limit the derivative of f at x_0.

Definition 1.1

The derivative of the function $f(x)$ with respect to the variable x is the function f' or $\frac{df}{dx}$ whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Example 1.2

Use the above definition to find the derivative of $f(x) = x^2$

We know that

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Here $f(x+h) = (x+h)^2 = x^2 + h^2 + 2hx$ so

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x^2 + h^2 + 2hx) - x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(h+2x)}{h} = \lim_{h \to 0} (h + 2x) = 2x$$