

MA140-Engineering Calculus

Lecture 8

September 22, 2017

A continuous function is one with an "unbroken" graph

Definition 1.1

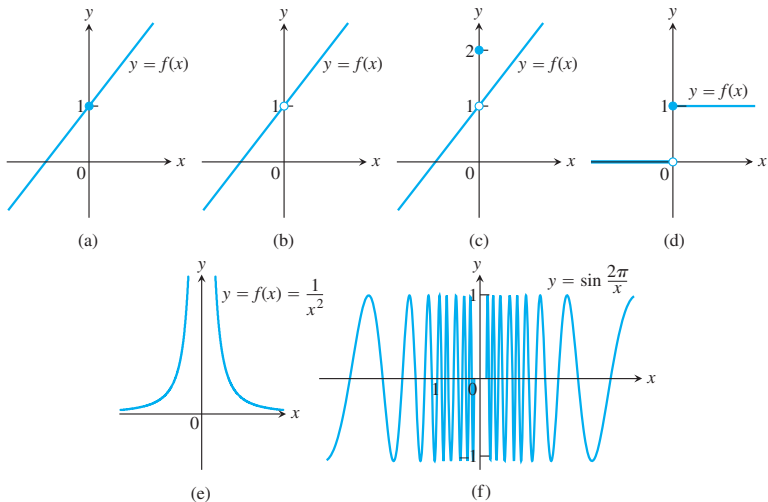
A function f is continuous at $x = a$ if:

- the point a is in the domain of f
- $f(x)$ has a limit as $x \rightarrow a$
- $\lim_{x \rightarrow a} f(x) = f(a)$

If f is continuous at every point in its domain, we say f is continuous.

Note: Lots of functions are continuous e.g. polynomials, trigonometric (not \tan), $|x|$ and so on.

Here are some examples of discontinuity.



Example 1.2

Consider the function

$$f(x) = \begin{cases} x + 1, & x < 2 \\ bx^2, & x \geq 2 \end{cases}$$

For what values of b is f continuous at $x = 2$.

We have

$$\lim_{x \rightarrow 2^-} f(x) = 2 + 1 = 3$$

and

$$\lim_{x \rightarrow 2^+} f(x) = b(2)^2 = 4b$$

Also note

$$f(2) = b(2)^2 = 4b$$

First, for the function to have a limit at $x = 2$,

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^+} f(x)$$

so $3 = 4b$, then $b = 3/4$, so

$$\lim_{x \rightarrow 2} f(x) = 3$$

Second, we will check that this value of b ensures

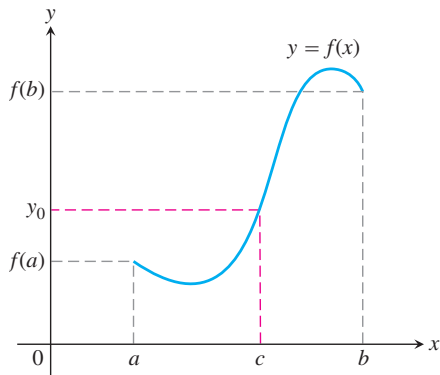
$$\lim_{x \rightarrow 2} f(x) = f(2)$$

$3 = 4b$ or $3 = 4(3/4)$. So when $b = 3/4$, $f(x)$ is continuous at $x = 2$.

(The Intermediate Value Theorem)

Theorem 1.3

A function $y = f(x)$ that is continuous on an interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$



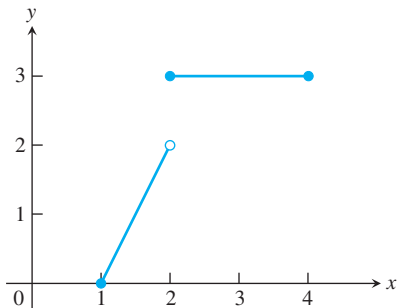
Example 1.4

Sketch a discontinuous graph for which the above theorem does not hold

The function

$$f(x) = \begin{cases} 2x - 2, & 1 \leq x < 2 \\ 3, & 2 \leq x \leq 4 \end{cases}$$

This function does not take on all values between $f(1) = 0$ and $f(4) = 3$, it misses all the values between 2 and 3.



Example 1.5

Show that there is a root of the equation

$$4x^3 - 6x^2 + 3x - 2 = 0$$

between 1 and 2.

Let $f(x) = 4x^3 - 6x^2 + 3x - 2$, we will use the above theorem with $a = 1, b = 2, c = 0$, so

- $f(x)$ is a polynomial so it is continuous
- $f(1) = 4 - 6 + 3 - 2 = -1 < 0$
 $f(2) = 32 - 24 + 6 - 2 = 12 > 0$
- and $f(a) < c < f(b)$

All conditions hold so, there exists a $y, 1 < y < 2$ such that $f(y) = 0$
So there is a root between 1 and 2

Example 1.6

How many roots does $x^3 + 1 = 3x^2$ have?

So,

$$x^3 - 3x^2 + 1 = 0$$

We define

$$f(x) = x^3 - 3x^2 + 1$$

$f(x)$ is a polynomial and hence continuous.

- $f(-1) = -1 - 3 + 1 < 0$ negative
- $f(0) = 1 > 0$ positive
- $f(2) = 8 - 3(4) + 1 < 0$ negative
- $f(3) = 1 > 0$ positive

So using The Intermediate Value Theorem, we see that $f(x) = 0$ has 3 roots.

Example 1.7

Find the number of asymptotes and discontinuities of

$$f(x) = \frac{(x-1)(x-2)}{(x+1)(x-3)}$$

Then find the x -intercept and y -intercept and plot f

Example 1.8

How many discontinuities has

$$f(x) = \begin{cases} \frac{(x-1)(x-2)}{(x+1)(x-3)}, & x \neq 1 \\ 1, & x = 1 \end{cases}$$