MA140-Engineering Calculus

Lecture 4

September 14, 2017

Example 1.1

What is the difference between these two functions:

$$\frac{x^2-1}{x-1} \quad and \quad x+1$$

we simplify $\frac{x^2-1}{x-1}$

$$\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1}$$

now the functions seem to be equal, the only difference between them is that tha rational function is undefined at $x = 1$, so

$$\frac{x^2-1}{x-1} = x+1 \quad for \quad x \neq 1$$

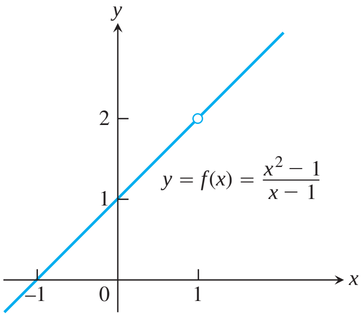
Example 1.2

How does the function

$$f(x) = \frac{x^2 - 1}{x - 1}$$

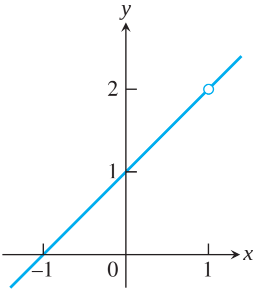
behave near $x = 1$

We have seen that the function f equals $x + 1$ except at $x = 1$, so the graph of f is thus the line $x + 1$ with the point $(1, 2)$ removed.

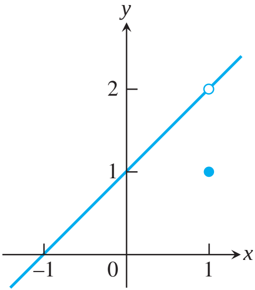


Values of x below and above 1	$f(x) = \frac{x^2 - 1}{x - 1} = x + 1, \quad x \neq 1$
0.9	1.9
1.1	2.1
0.99	1.99
1.01	2.01
0.999	1.999
1.001	2.001
0.999999	1.999999
1.000001	2.000001

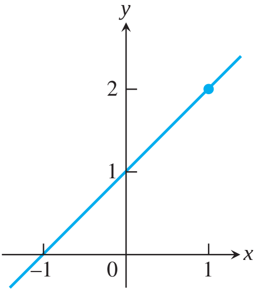
$$\lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$



(a) $f(x) = \dfrac{x^2 - 1}{x - 1}$



(b) $g(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$



(c) $h(x) = x + 1$

Finding the Limits

Sometimes $\lim_{x \to a} f(x)$ can be evaluated by calculating $f(a)$. This holds, for example, whenever $f(x)$ is an algebraic combination of polynomials and trigonometric functions for which $f(a)$ is defined.

- $\lim_{x \to 3} x = 3$
- $\lim_{x \to 3} 5 = 5$
- $\lim_{x \to 3} (5x - 2) = 13$

Note: The Identity and Constant Functions Have Limits at Every Point.

The Limit Laws

If L, M, a and k are real numbers and

$$\lim_{x \to a} f(x) = L \quad and \quad \lim_{x \to a} g(x) = M$$

then:

- *Sum Rule*: The limit of the sum of two functions is the sum of their limits.

$$\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M$$

- *Difference Rule:* The limit of the difference of two functions is the difference of their limits.

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M$$

- *Product Rule:* The limit of a product of two functions is the product of their limits.

$$\lim_{x\to a}[f(x)\cdot g(x)] = (\lim_{x\to a} f(x))\cdot(\lim_{x\to a} g(x)) = L\cdot M$$

- *Constant Multiple Rule:* The limit of a constant times a function is the constant times the limit of the function.

$$\lim_{x\to a}(k\cdot f(x)) = k\cdot(\lim_{x\to a} f(x) = k\cdot L$$

- *Quotient Rule:* The limit of a quotient of two functions is the quotient of their limits, provided the limit of the denominator is not zero.

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{L}{M}, \quad M\neq 0$$

- *Power Rule:* If r and s are integers with no common factor and $s \neq 0$, then

$$\lim_{x \to a} (f(x))^{r/s} = (\lim_{x \to a} f(x))^{r/s} = L^{r/s}$$

Example 1.3

- $\lim_{x\to 1}(x^3 + 4x^2 - 3) = \lim_{x\to 1} x^3 + \lim_{x\to 1} 4x^2 - \lim_{x\to 1} 3 = 1 + 4 - 3 = 2$

Example 1.4

- $\lim_{x \to 1} \dfrac{x^4 + x^2 - 1}{x^2 + 5} = \dfrac{\lim_{x\to 1}(x^4 + x^2 - 1)}{\lim_{x\to 1}(x^2 + 5)} = \dfrac{1}{6}$

Example 1.5

- $\lim_{x\to 1} \sqrt{4x^2 - 3} = \sqrt{\lim_{x\to 1}(4x^2 - 3)} = 1$