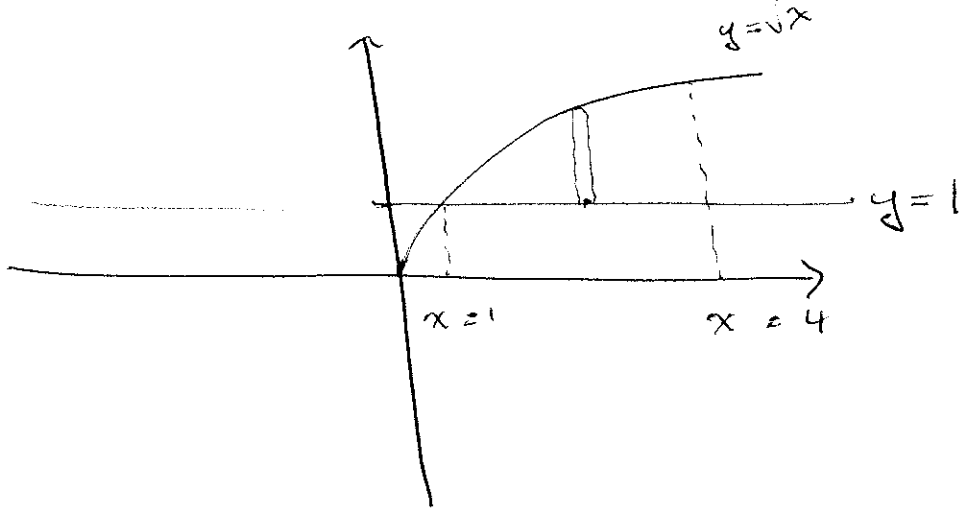
① Find the volume of the solid generated by revolving the region bounded by $\sqrt{x}$ and the lines $y=1$, $x=4$ about the line $y=1$.



$$V = \int_a^b \pi (R(x))^2 \, dx = \int_1^4 \pi (\sqrt{x} - 1)^2 \, dx = \cdots$$

② Sketch the region below the graph of the curve $y(x) = \frac{1}{2x-1}$ and above the x-axis for $x \in [3,4]$. Now compute the volume of the solid obtained by rotating this region about the x-axis.

$$y(x) = \frac{1}{2x-1} \Rightarrow y'(x) = \frac{-2}{(2x-1)^2} < 0$$

$$\Rightarrow y''(x) = \frac{4(2x-1)}{(2x-1)^4} = \frac{4}{(2x-1)^3}$$

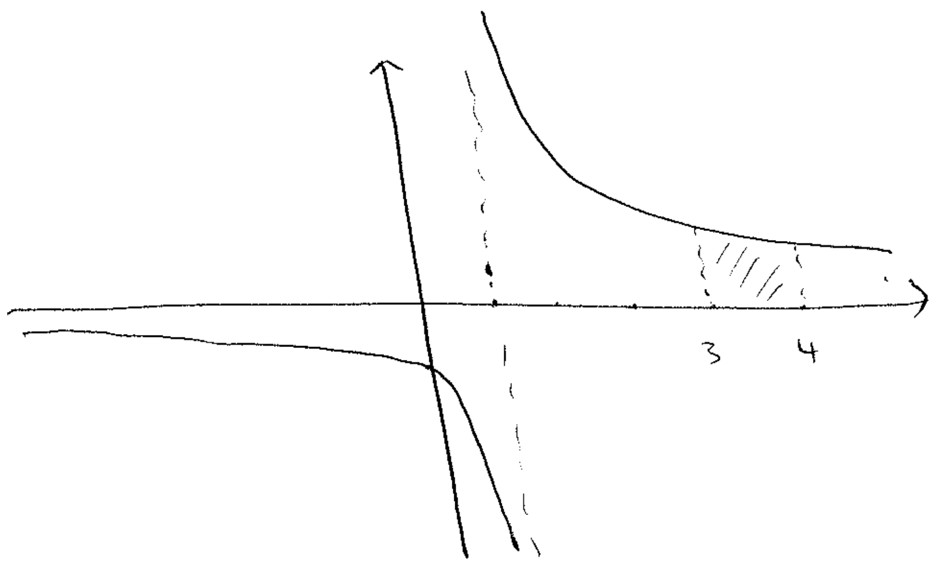
$$\lim_{x \to \infty} \frac{1}{2x-1} = 0 \Rightarrow y = 0 \text{ is the horizontal asymptote}$$

$$2x - 1 = 0 \Rightarrow x = \frac{1}{2} \text{ is the vertical asymptote}$$

$$\lim_{x \to \frac{1}{2}^+} \frac{1}{2x-1} = +\infty \quad , \quad \lim_{x \to \frac{1}{2}^-} \frac{1}{2x-1} = -\infty$$

$\Rightarrow$

		$\frac{1}{2}$	
$f'(x)$	$-$		$-$
$f''(x)$	$-$		$+$
$f(x)$	$\searrow$ $\cap$	undefined	$\searrow$ $\cup$



$x = \frac{1}{2}$

so $R(x) = f(x) = \frac{1}{2x-1}$

then:

$$V = \int_3^4 \pi \left(\frac{1}{2x-1}\right)^2 dx = \cdots$$

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③ Evaluate the following integral:

$$\int \ln x \, dx = \int u \, dv = uv - \int v \, du \quad (*)$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

so $(*) = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$