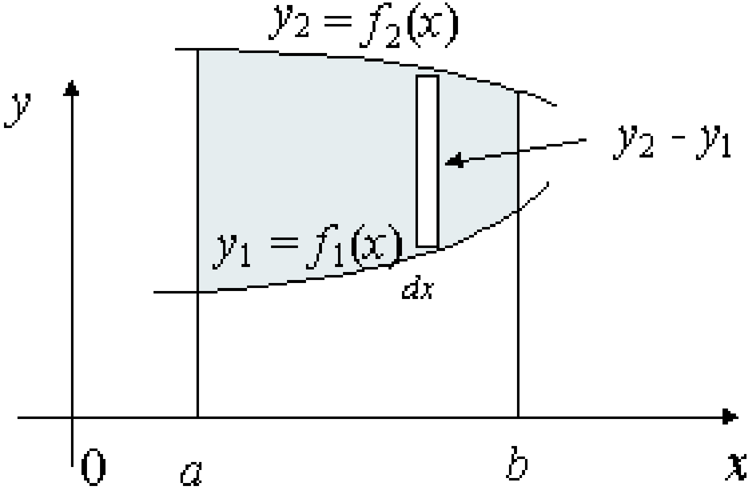
MA140-Engineering Calculus

Lecture 32

November 15, 2017

Centroid of a plane area:



Suppose the centroid of a typical strip is at (x_k, y_k) Then:

$$System\ \ mass \cong \sum_{k=1}^{n} \Delta m_k$$

The moments of the entire system about the two axes are:

$$Moment\ \ about\ \ x - axis \cong \sum_{k=1}^{n} y_k \Delta m_k$$

$$Moment\ \ about\ \ y - axis \cong \sum_{k=1}^{n} x_k \Delta m_k$$

The x-coordinate of the system's center of mass is defined to be:

$$\bar{x} = \frac{Moment\ \ about\ \ y - axis}{System\ \ mass} \cong \frac{\sum_{k=1}^{n} x_k \Delta m_k}{\sum_{k=1}^{n} \Delta m_k}$$

As $\delta = 1$, $\Delta m_k = \Delta A_k = (f_2(x_k) - f_1(x_k))\Delta x$. So:

$$\bar{x} \cong \frac{\sum_{k=1}^{n} x_k (f_2(x_k) - f_1(x_k))\Delta x}{\sum_{k=1}^{n} (f_2(x_k) - f_1(x_k))\Delta x}$$

The sums is a Riemann sum so when $\Delta x \to 0$ or $n \to \infty$, then

$$\bar{x} = \frac{\int_a^b x(f_2(x) - f_1(x))dx}{\int_a^b (f_2(x)) - f_1(x))dx} = \frac{\int_a^b x(f_2(x) - f_1(x))dx}{A}$$

The y-coordinate of the system's center of mass is defined to be:

$$\bar{y} = \frac{1}{2A} \int_a^b \left[(f_2(x))^2 - (f_1(x))^2 \right] dx$$

Example 1.1

Find the centroid of the area under the curve $y = \sqrt{x - 2}$ between the lines $x = 2$ and $x = 5$

$$Area = \int_2^5 \sqrt{x-2}dx = \int_2^5 (x-2)^{1/2}dx = \frac{(x-2)^{3/2}}{3/2}\Big]_2^5$$

$$= \frac{2}{3}[(5-2)^{3/2} - (2-2)^{3/2}] = \frac{2}{3}3\sqrt{3} = 2\sqrt{3}$$

the x coordinate of the centroid is:

$$\bar{x} = \frac{1}{A}\int_2^5 x\sqrt{x-2}dx$$

Let $u = x - 2$, so $x = u + 2$ and $du = dx$ then:

$$\int x\sqrt{x-2}dx = \int (u+2)\sqrt{u}du = \int u^{3/2} + 2u^{1/2}du = \frac{u^{5/2}}{5/2} + 2\frac{u^{3/2}}{3/2} + c$$

$$= \frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2} + c$$

$$\bar{x} = \frac{1}{A}\int_2^5 x\sqrt{x-2}dx = \frac{1}{2\sqrt{3}}\Big(\frac{2}{5}(x-2)^{5/2} + \frac{4}{3}(x-2)^{3/2}\Big]_2^5\Big) = \frac{19}{5}$$

Also:

$$\bar{y} = \frac{1}{2A}\int_2^5 [f(x)]^2 dx = \frac{1}{2A}\int_2^5 (x-2)dx = \frac{1}{2A}(\frac{x^2}{2} - 2x\Big]_2^5)$$

So

$$\bar{y} = \frac{3\sqrt{3}}{8}$$