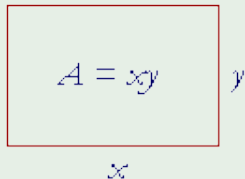


## Solving Applied Optimization Problems:

- (1) *Read the problem.* Read the problem until you understand it. What is given? What is the unknown quantity to be optimized?
- (2) *Draw a picture.* Label any part that may be important to the problem.
- (3) *Introduce variables.* List every relation in the picture and in the problem as an equation or algebraic expression, and identify the unknown variable.
- (4) *Write an equation for the unknown quantity.* If you can, express the unknown as a function of a single variable or in two equations in two unknowns. This may require considerable manipulation.

## Example 1.1

A rectangle has the following side lengths



Find  $x$  and  $y$  if the area is to be maximized if the perimeter equals 30m.

The perimeter equals  $30m$ , it means  $2(x + y) = 30$  or  $x + y = 15$ , so

$$y = 15 - x, (\star)$$

Area =  $A = x \cdot y$ , Using  $(\star)$ , we have

$$A = x(15 - x)$$

Now  $A$  is a function of  $x$ , in order to find the maximum value of  $A$ , we differentiate it:

$$\frac{dA}{dx} = 15 - 2x$$

Let:

$$\frac{dA}{dx} = 0 \Rightarrow x = \frac{15}{2} (y = \frac{15}{2})$$

So  $x = \frac{15}{2}$  is a critical point.

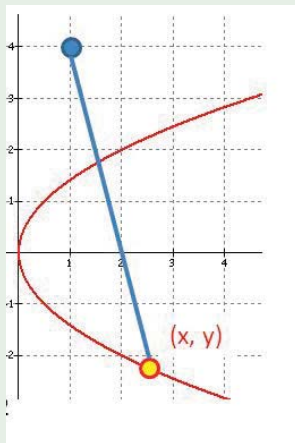
Now we use the second derivative test

$$\frac{d^2 A}{dx^2} = 2 < 0$$

so  $x = \frac{15}{2}$  is a local maximum

## Example 1.2

Find the point on the parabola  $y^2 = 2x$ , closest to the point  $(1, 4)$



The distance between the two points  $(x, y)$  and  $(1, 4)$  equals:

$$d = \sqrt{(x - 1)^2 + (y - 4)^2}$$

Instead of minimizing  $d$  we can minimize distance squared.

$$r = (x - 1)^2 + (y - 4)^2$$

As the coordinates of the point satisfy  $y^2 = 2x$  or  $\frac{y^2}{2} = x$ , let  $y = t$  then  $x = \frac{t^2}{2}$ . So  $(x, y) = (\frac{t^2}{2}, t)$  and  $(1, 4)$  have distance squared equal to

$$r(t) = \left(\frac{t^2}{2} - 1\right)^2 + (t - 4)^2$$

$$r(t) = \frac{t^4}{4} - t^2 + 1 + t^2 + 16 - 8t = \frac{t^4}{4} - 8t + 17$$
$$r'(t) = t^3 - 8 = 0$$

So  $t = 2$  is the only critical point.

$$r''(t) = 3t^2 \Rightarrow r''(2) = 12 > 0$$

So  $t = 2$  is a local minimum, therefore  $(2, 2)$  is the point on  $y^2 = 2x$  closest to  $(1, 4)$