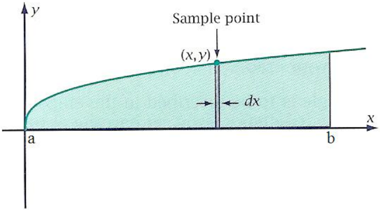
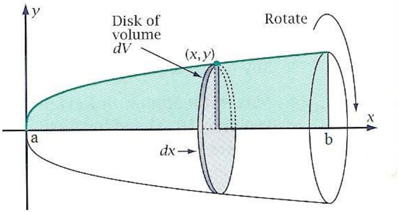
Find the volume of solids:





The solid generated by rotating a plane region about an axis in its plane is called a solid of revolution. To find the volume of a solid we need only observe that the cross-sectional area $A(x)$ is the area of a disk of radius $R(x)$, the distance of the planar regions boundary from the axis of revolution. The area is then

$$A(x) = \pi(radius)^2 = \pi[R(x)]^2$$

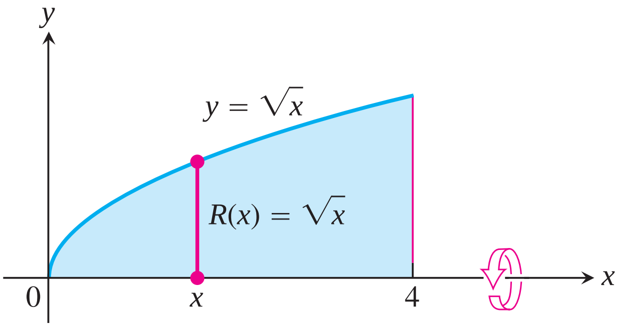
So the definition of volume gives:

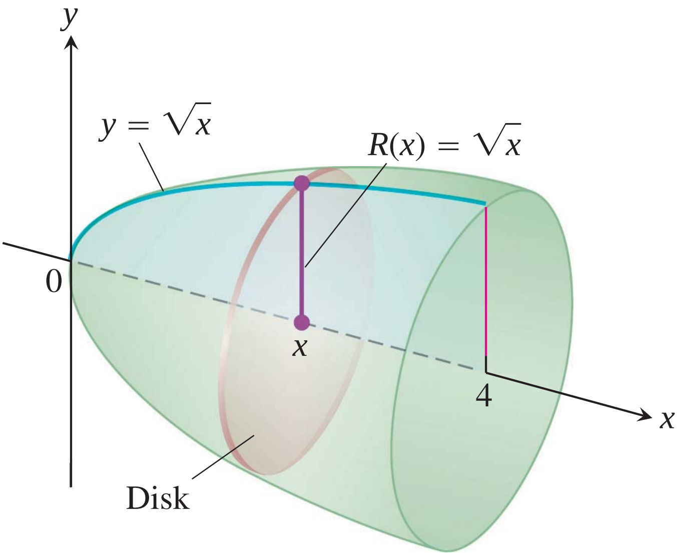
$$V = \int_a^b A(x)dx = \int_a^b \pi[R(x)]^2 dx$$

Example 1.1

The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$, and the $x-$axis is revolved about the $x-$axis to generate a solid. Find its volume.

We draw figures showing the region, a typical radius, and the generated solid.





y
$y = \sqrt{x}$
$R(x) = \sqrt{x}$
0
x
4
x
Disk

The volume is:

$$V = \int_a^b \pi [R(x)]^2 dx == \int_0^4 \pi [\sqrt{x}]^2 dx$$

$$= \pi \int_0^4 x dx = \pi \frac{x^2}{2}\Big]_0^4 = \pi \frac{(4)^2}{2} = 8\pi$$

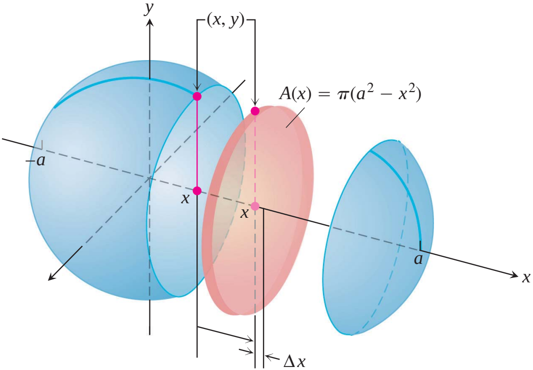
Example 1.2

The circle

$$x^2 + y^2 = a^2$$

is rotated about the $x-$axis to generate a sphere. find its volume

We imagine the sphere cut into thin slices,



the cross-sectional area at a typical point x between $-a$ and a is:

$$A(x) = \pi y^2 = \pi(a^2 - x^2)$$

Therefore, the volume is

$$V = \int_{-a}^{a} A(x)dx = \int_{-a}^{a} \pi(a^2 - x^2)dx$$

$$= \pi\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a} = \frac{4}{3}\pi a^3$$

Example 1.3

Sketch the area between $x = 2$ and $x = 3$, under the curve

$$y = \frac{1}{x - 1}$$

Also find the volume of the solid if this area is rotated around the $x-$axis.