MA140-Engineering Calculus

Lecture 16

October 23, 2019

Tangents

Example 1.1

Determine the (equation of the) tangent (line) to the function

$$f(x) = x^2$$

at the point $x = 5$.

$$\begin{aligned} f(x) = x^2 &\implies f'(x) = 2x \\ \text{At the point } x = 5, y = f(x) = f(5) = 5^2 &= 25. \\ \text{At the point } x = 5, f'(x) = f'(5) = (2)(5) &= 10. \end{aligned} \tag{1}$$

This means, the tangent line has slope 10 and goes through the point $(5, 25)$. The equation of a line with slope m is $y = mx + c$. So here we have $y = 10x + c$. Since the line must go through $(5, 25)$, we insert this point in to the equation to get $25 = (10)(5) + c \implies c = -25$. Finally our answer is $y = 10x - 25$.

Example 1.2

Find the points on the curve $y = 4x^3 - 6x$ where the tangent (line) is parallel to $2x + y + 5 = 0$.

To say two lines are "parallel" means they have the same slope:

1. What is the slope of the line? Rewrite it as $y = -2x - 5$, so the slope is -2. (The slope of a line $y = mx + c$ is m.)
2. What is the slope at any point on the curve? $y = 4x^3 - 6x \implies y' = 12x^2 - 6$.

So we just set these two slopes equal to one another:
$-2 = 12x^2 - 6 \implies 4 = 12x^2 \implies x = 1/\sqrt{3}$ or $x = -1/\sqrt{3}$.

- $x = 1/\sqrt{3} \implies y = 4(1/\sqrt{3})^3 - 6/\sqrt{3} = -14/(3\sqrt{3})$
- $x = -1/\sqrt{3} \implies y = -4(1/\sqrt{3})^3 + 6/\sqrt{3} = 14/(3\sqrt{3})$

Example 1.3

Suppose a mug has the shape of a cylinder, and is being filled with water. At a particular point in time, the height of the water is rising at a rate of 1cm per second. If the circumference of the mug is 6π cm, what is the rate of increase of (the volume of) water in the mug?

$V = \pi r^2 h$. Circumference $= 2\pi r = 6\pi \implies r = 3 \implies V = 9\pi h$.

$$\frac{dV}{dt} = 9\pi \frac{dh}{dt} = 9\pi(1) = 9\pi \text{ (cubic cemtimetres per second)}$$