MA140-Engineering Calculus

Lecture 22

October 24, 2017

Example 1.1

Evaluate

$$\int x^2 e^x dx$$

With

$$u = x^2, \quad dv = e^x dx$$

$$du = 2x dx, \quad v = e^x$$

Using integration by parts,

$$\int u dv = uv - \int v du$$

We get:

$$\int x^2 e^x dx = x^2 e^x - 2 \int x e^x dx$$

$$\int xe^x dx = ?$$

we have to use integration by parts more than once. We integrate by parts again with

$$u = x, \quad dv = e^x dx$$

$$du = dx, \quad v = e^x$$

So

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + c$$

Hence:

$$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx = x^2 e^x - 2xe^x + 2e^x + c$$

Example 1.2

Evaluate:

$$\int e^x \cos(x)dx$$

Let

$$u = e^x, \quad dv = \cos(x)dx$$

Then:

$$du = e^x dx, \quad v = \sin(x)$$

Using integration by parts,

$$\int udv = uv - \int vdu$$

We get:

$$\int e^x \cos(x)dx = e^x \sin(x) - \int e^x \sin(x)dx$$

The second integral is like the first except that it has $\sin x$ in place of $\cos x$. To evaluate it, we use integration by parts with:

$$u = e^x, \quad dv = \sin(x)dx$$

$$v = -\cos(x), \quad du = e^x dx$$

Then:

$$\int e^x \cos(x)dx = e^x \sin(x) - \Big(-e^x \cos(x) - \int (-\cos(x))(e^x dx) \Big)$$

$$= e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)dx$$

The unknown integral now appears on both sides of the equation. Adding the integral to both sides and adding the constant of integration gives:

$$2\int e^x \cos(x)dx = e^x \sin(x) + e^x \cos(x) + c$$

Dividing by 2 and renaming the constant of integration gives:

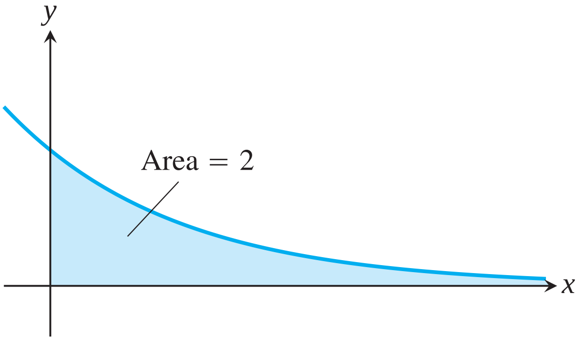
$$\int e^x \cos(x)dx = \frac{e^x \sin(x) + e^x \cos(x)}{2} + c$$

Infinite Limits of Integration:

Example 1.3

Evaluate:

$$\int_0^\infty \frac{1}{e^{\frac{x}{2}}}dx$$



First find the area $A(b)$ of the portion of the region that is bounded on the right by $x = b$.

$$A(b) = \int_0^b \frac{1}{e^{\frac{x}{2}}} dx = \int_0^b e^{-x/2} dx = -2e^{-x/2}\Big]_0^b = -2e^{-b/2} + 2$$

Then find the limit of $A(b)$ as $b \to \infty$:

$$\lim_{b\to\infty} A(b) = \lim_{b\to\infty} (-2e^{-b/2} + 2) = 2$$

Inverse Trigonometric Functions:

- $$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}(x) + c$$
- $$\int \frac{dx}{1+x^2} = \tan^{-1}(x) + c$$
- $$\int \frac{dx}{\sqrt{x^2-1}} = \sec^{-1}(x) + c$$