Searching

We have a list (array) L of n items, $L(1), L(2), L(3), \ldots L(n)$. For example, if $L = [5, 27, 13, 71]$ then $n = 4$ and $L(1) = 5, L(2) = 27, L(3) = 13, L(4) = 71$. We want to search this list for some item X. Of course X may not be in the list at all, it may be in the list once, or it may be in the list multiple times.

Linear (Sequential) Search Just search from left to right in the list:

1. Compare X with $L(1)$
2. Compare X with $L(2)$
3. Compare X with $L(3)$
4. ...
5. Compare X with $L(n)$

If we know X occurs in the list once, then on average we will have to look through $n/2$ items to find it (worst case: n comparisons, best case: 1 comparison). So this search is $\mathbf{O}(n)$.

Binary Search **This search algorithm only applies to ordered (sorted) data**

1. Compare X to the middle value in the list (i.e. $L(n/2)$).
2. If $X = L(n/2)$, we are finished, terminate the algorithm with "X found".
3. If $X > L(n/2)$ do a binary search to the right of the mid-point (i.e. a binary search of the elements $L(n/2+1), L(n/2+2), \ldots L(n)$), from step 1.
4. If $X < L(n/2)$ do a binary search to the left of the mid-point (i.e. a binary search of the elements $L(1), L(2), \ldots L(n/2-1)$), from step 1.

5. If in either step 3. or step 4. there is no “right” or “left” of the current point, terminate the algorithm with “X not found”.

Binary Search- Example 1

Search $L = [23, 27, 45, 47, 51, 86, 97]$ for the number $X = 51$

1.
   - $X = L(n/2)$?
   - i.e. $51 = L(4)$?
   - i.e. $51 = 47$? No!
2. $51 > 47$? Yes!
3. So do binary search to the right, of the list $[51, 86, 97]$
4. $51 = 86$? No!
5. $51 > 86$? No!
6. So do binary search to the left, i.e. binary search of the list $[51]$
7. $51 = 51$ Yes!
8. Terminate with “X found”.

Binary Search- Example 2

Search $L = [23, 27, 45, 47, 51, 86, 97]$ for the number $X = 50$

1.
   - i.e. $50 = L(4)$?
   - i.e. $50 = 47$? No!
2. $50 > 47$? Yes!
3. So do binary search to the right, of the list $[51, 86, 97]$
4. $50 = 86$? No!
5. $50 > 86$? No!
6. So do binary search to the left, i.e. binary search of the list $[51]$
7. $50 = 51$ No!
8. $50 > 51$ No!
9. So do binary search to the left: But there is no left.
10. Terminate with “X not found”.

Efficiency

Let $N(n)$ denote the number of comparisons we need to make in a binary search of a list of length n. After the first "iteration", we have made one comparison, and we have a (new) problem to do a binary search of a list of size (approx.) $n/2$, so we can write

$$N(n) \approx 1 + N(n/2)$$

We can re-substitute repeatedly in this equation (representing further iterations) to get

$$
\begin{aligned}
N(n) &= 1 + N(n/2) \\
&= 1 + [1 + N(\frac{n/2}{2})] \\
&= 2 + N(n/4) \\
&= 2 + [1 + N(\frac{n/4}{2})] \\
&= 3 + N(n/8) \\
&= 3 + [1 + N(\frac{n/8}{2})] \\
&= 4 + N(n/16) \\
&= \ldots \\
&= k + N(n/2^k)
\end{aligned}
$$

Clearly, $N(1) = 1$, so we terminate when

$$\frac{n}{2^k} = 1$$

or $k \approx \log_2 n$, which gives $N(n) \approx \log_2 n$. So the Binary Search Algorithm is $\mathbf{O}(\log_2 n)$.

Pseudocode

Linear Search

```
BEGIN
IN(L, n, X)
{Linear Search of a List L of length n
for item X}
FOR i ← 1 TO n DO
   IF L(i)=X THEN OUT(i)
END
```

OR

```
BEGIN
IN(L, n, X)
LS(L,1,n,X)
END

LS(M,i,j,Y)
{Linear Search of a List M between
index i and j for item Y}
IF i>j THEN OUT("Y not found")
  ELSE IF Y=M(i) THEN OUT(i)
    ELSE LS(M,i+1,j,Y)
```

Binary Search

```
BEGIN
IN(L, n, X)
BS(L,1,n,X)
END

BS(M,i,j,Y)
{Binary Search of a List M between
index i and j for item Y}
IF i>j THEN OUT("Y not found")
MP ← FLOOR((i+j)/2)
IF Y=M(MP) THEN OUT("Y found at MP")
  ELSE IF Y>M(MP) THEN BS(M,MP+1,j,Y)
    ELSE BS(M,i,MP-1,Y)
```

Aside

`FLOOR` truncates numbers to their integer value: e.g.

- `FLOOR(3)` = 3
- `FLOOR(3.4)` = 3
- `FLOOR(3.8)` = 3

You can regard this as "built in" to the pseudocode language, or if necessary it can be easily calculated:

```
FLOOR(x)
i ← 0
WHILE i≤ x DO i ← i+1
FLOOR ← i-1
```

Graphs

A graph is a set V of nodes (or vertices), along with a set E of edges (or arcs) connecting (some of) the vertices.

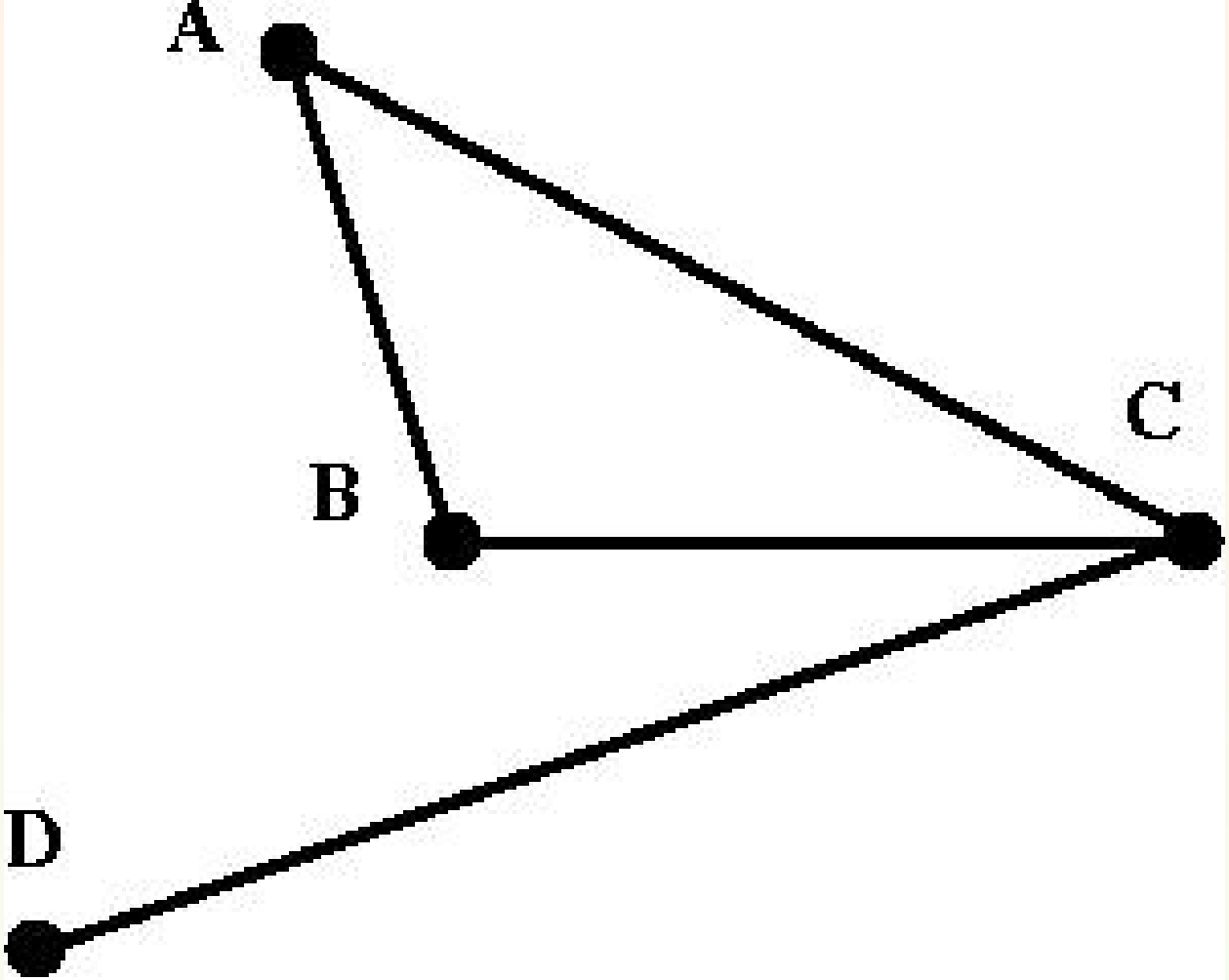
Example 1 $V = \{1, 2, 3, 4\},\ E = \{(1, 2), (2, 3), (1, 4)\}$ (The edges are just written as pairs of nodes that they connect).

Example 2 $V = \{u, f, s, o, p\},\ E = \{(o, s), (f, u), (p, o)\}$ (There is no concept of "direction" to the edge: i.e. (o,s) is the same as (s,o).)

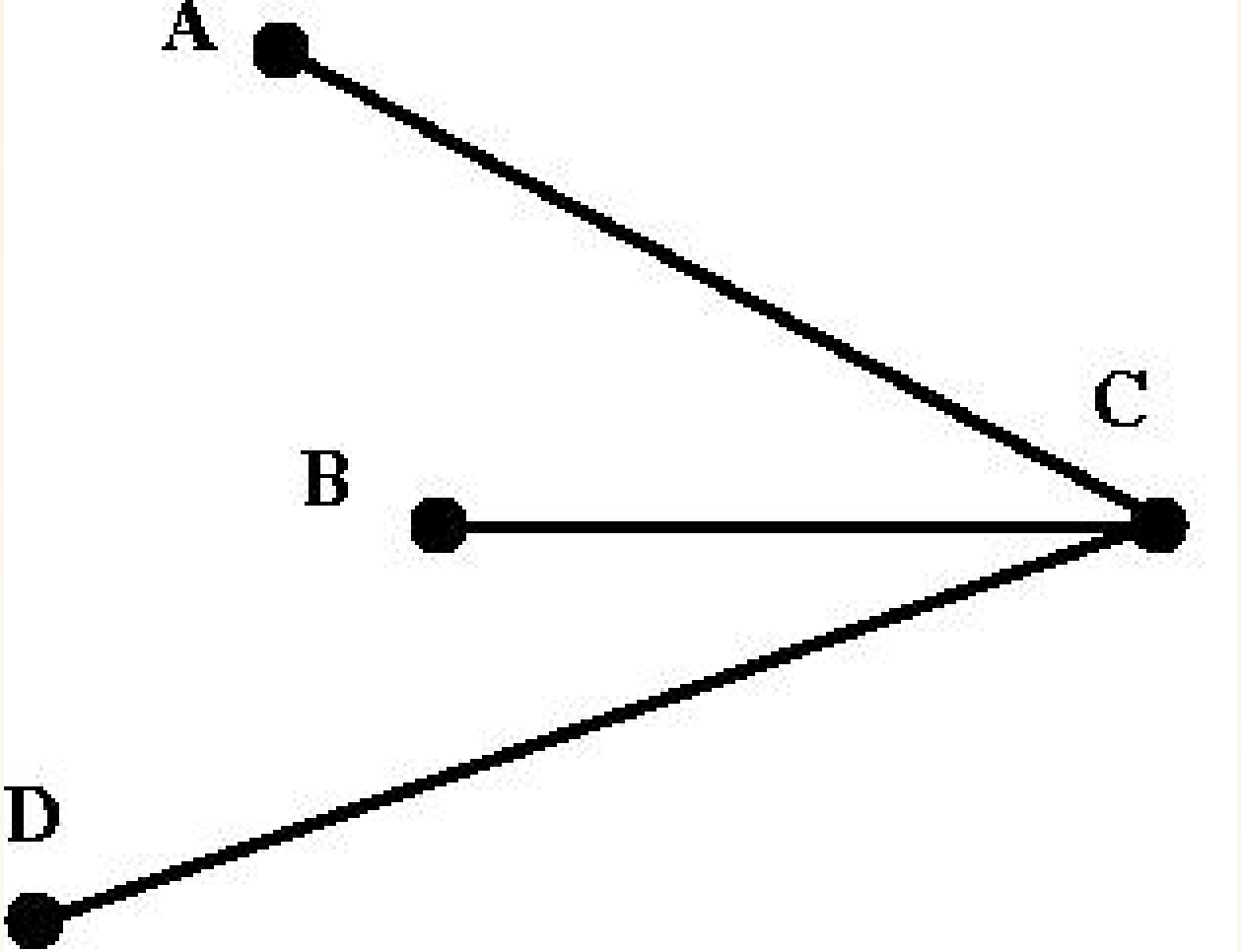
Example 2 $V = \{f, 5, up, red\},\ E = \{(5, up), (5, 5), (red, up), (red, 5)\}$

- We say there is a path from node v to node u on a graph if it is possible to travel from v to u (via intermediate nodes) along the edges.
- The length of a path from u to v is the number of edges in it.
- A cycle is a path from some node v to itself.
- A graph is said to be cyclic if there is at least one cycle in the graph.

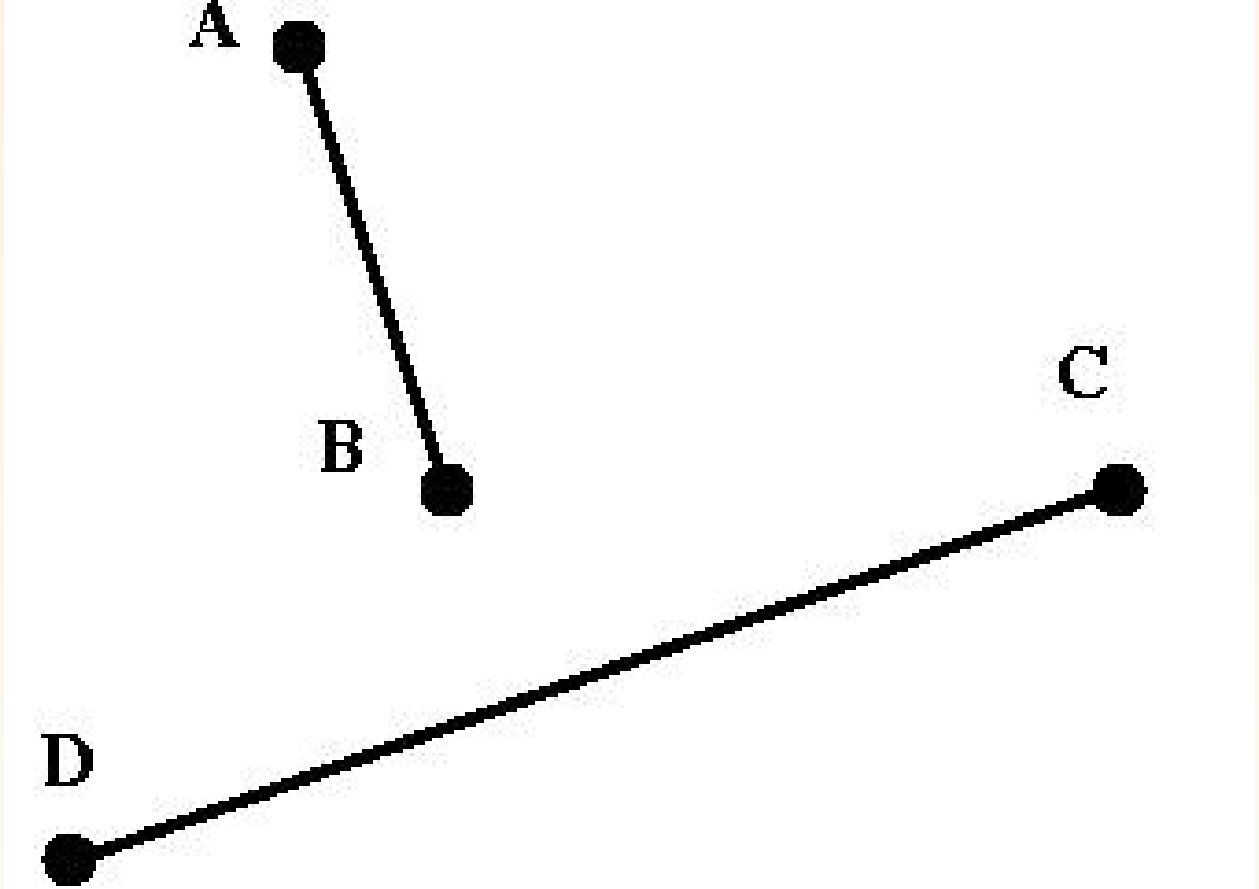
- A graph is said to be connected if there is at least one path between any two nodes in the graph.



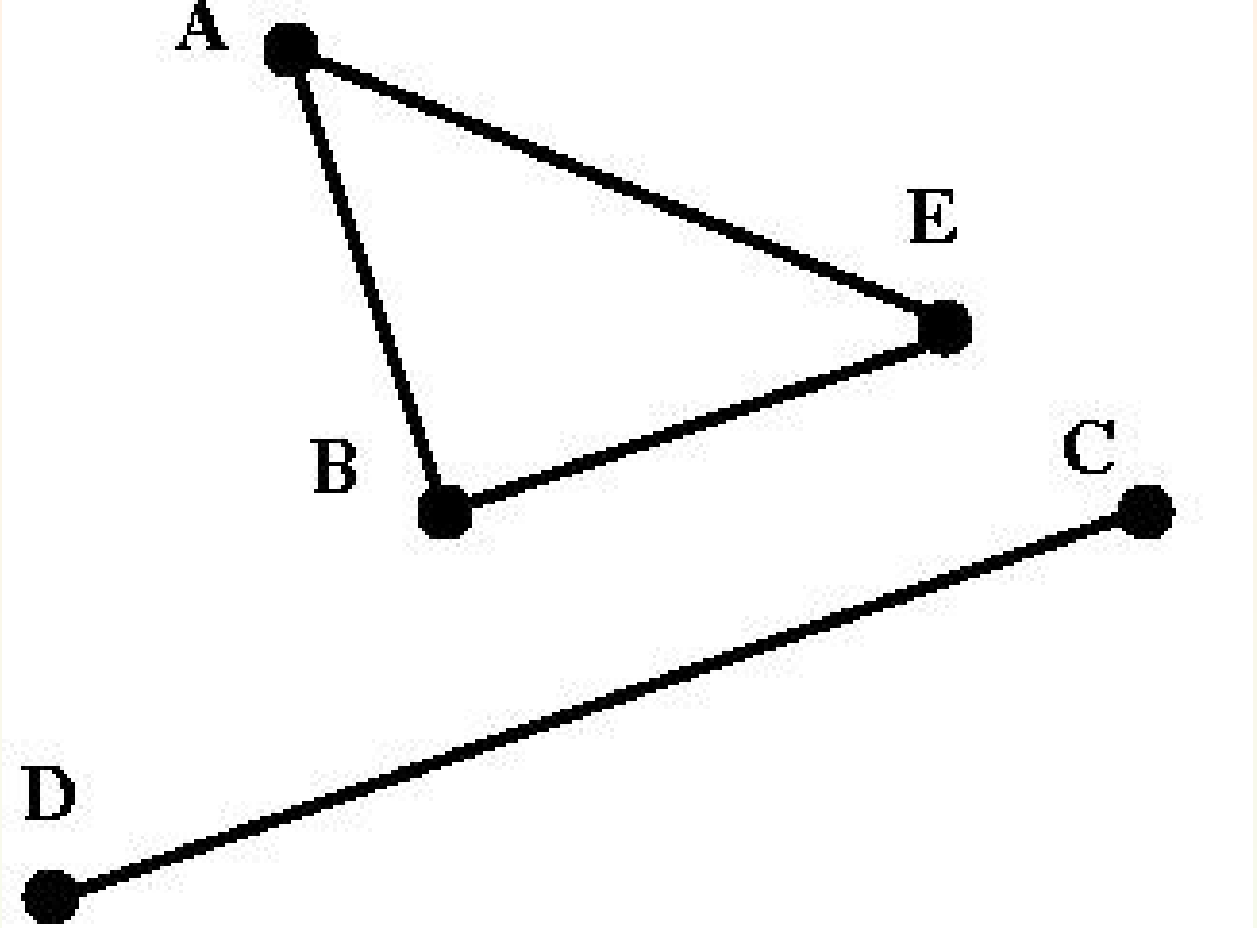
CYCLIC CONNECTED GRAPH



ACYCLIC CONNECTED GRAPH



ACYCLIC DISCONNECTED GRAPH



CYCLIC DISCONNECTED GRAPH

A tree is an acyclic connected graph. In a tree, we designate one node (any node) to be the root node. This is conventionally drawn at the top of the tree: all other nodes are measured by their distance from the root.

1. Because a tree is connected we know there is a path from every node to the root.
2. Because a tree is acyclic we know that there is only one path to the root.
3. Note that an acyclic disconnected graph is known as a forest: it is composed of many trees.
4. The parent of a given node v is the node that is connected directly to v but is closer to the root. (Thus the root has no parent).
5. u is a parent of v if and only if v is a child of u.