Lecture 3: Functions ctd. / Polynomials

MA140: Engineering Calculus.

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Functions

Recall from yesterday:

Example

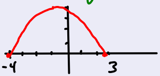
Identify the domain, codomain and range of

1. $f(x) = 3x^2 + 1$
2. $f(x) = \sqrt{(x+4)(3-x)}$
3. $g(x) = \frac{1}{x}$

Solution ctd.

(2).

- Sketch a graph.



Lowest value is 0 (at $x=-4$ and $x=3$)
Highest value is $\frac{7}{2} = 3.5$ at $x = -\frac{1}{2}$
So, range $= [0, \frac{7}{2}]$

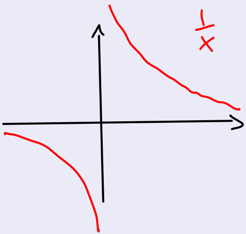
- We can take $\mathbb{R}$ to be the codomain.

Functions

Solution ctd.

③ $g(x) = \frac{1}{x}$

- Domain $= \mathbb{R} \setminus \{0\}$
- Range $= \mathbb{R} \setminus \{0\}$
- Codomain $= \mathbb{R}$



Note: Geogebra - App, Wolfram Alpha, etc. might be helpful.

A Catalog of Functions.

There are many **different types of functions** that can be used to **model relationships** between objects in the **real world**. The most common essential types of functions are:

- **Linear** Functions,
- **Polynomial** Functions,
- **Power** Functions,
- **Rational** Functions,
- **Algebraic** Functions,
- **Trigonometric** Functions,
- **Exponential** Functions,
- **Logarithms**.

Polynomials

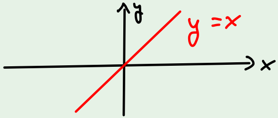
A **polynomial function** (or just **polynomial**) is a function of the form

$$y = f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0, \quad x \in \mathbb{R},$$

where $a_0, a_1, \ldots, a_n$ are real numbers. They are called **coefficients** of the polynomial. The number **n** is called the **degree** of the polynomial.

Example

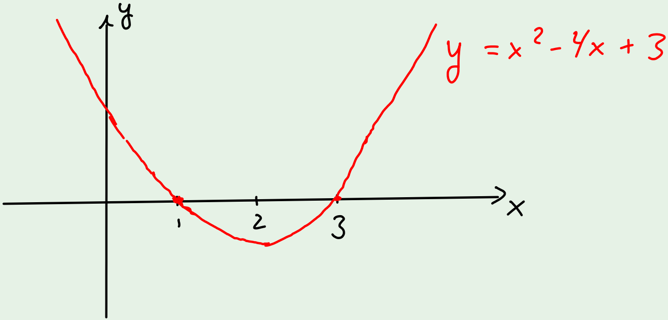
$y = x$ is a **linear** polynomial with degree $n = 1$.



Polynomials

Example

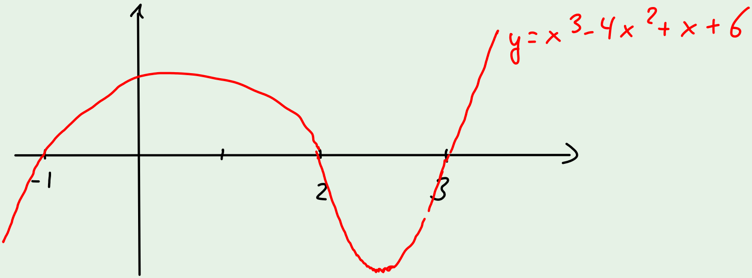
$y = x^2 - 4x + 3$ is a **quadratic** polynomial with degree $n = 2$.



Polynomials

Example

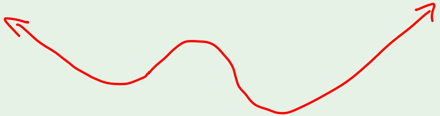
$y = x^3 - 4x^2 + x + 6$ is a **cubic** function with degree $n = 3$.



Sketching the graph of a polynomial function

Fact: A polynomial function of grade n has **up to** $n-1$ bends,

e.g. a typical fourth degree polynomial has 3 bends.



Exercise

Make a plan: what do you have to do to sketch the graph of

$$y = x^3 - 4x^2 + x + 6 \quad ?$$

Sketching the graph of a polynomial function

- To sketch the graph, first find the **intercepts**:
  - The **y-intercepts** can be found by letting $x = 0$.
  - The **x-intercepts** are called the **roots** (or **zeros**).
    To find the roots, set y equal to zero and solve for x.
- You don't have to use the same scale on the x- and on the y-axis.
- Do not use graph paper.

Example

Sketch the graph of

$$y = x^3 - 4x^2 + x + 6$$

Sketching the graph of a polynomial function

Solution

$y = x^3 - 4x^2 + x + 6$

(polynomial of grade 3 $\rightarrow$ 2 bends)

Find y - intercept.

$x = 0 \Rightarrow y = 0^3 - 4(0^2) + 0 + 6 = 6$

So $(0, 6)$ is y - intercept.

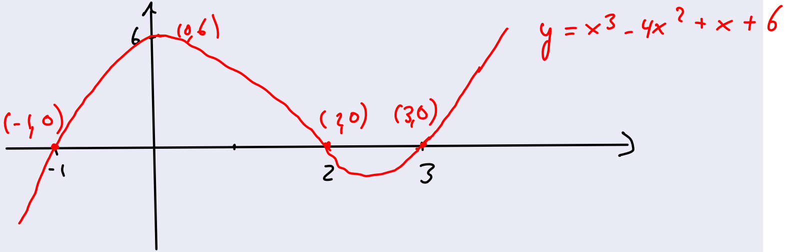
Find x - intercepts

$y = 0 \Rightarrow 0 = x^3 - 4x^2 + x + 6$

By trial we find that $x = -1, 2, 3$ are roots.

Sketching the graph of a polynomial function

Solution ctd.



Exercises

Sketch the graphs of

(i) $y = 5x^2 - 7$

(ii) $f(x) = x^2 - 4x + 3$

(iii) $y = x^3 - 6x^2 - 11x - 6$

Exercises

Sketch the graph of

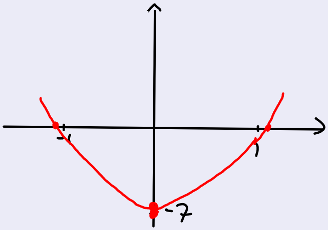
(i) $y = 5x^2 - 7$

Solution

(i) $y = 5x^2 - 7$ (up to 1 bend)

$x = 0 \Rightarrow y = 5(0^2) - 7 = -7$,
so $(0, -7)$ is y-intercept

$y = 0 \Rightarrow 0 = 5x^2 - 7$
$x^2 = 7/5$
$\Rightarrow x = \pm\sqrt{\frac{7}{5}} \approx \pm 1.18$,
so $(-1.18, 0)$, $(1.18, 0)$ are x-intercepts



Exercises

Sketch the graph of

(ii) $f(x) = x^2 - 4x + 3$

Solution

(ii) $f(x) = x^2 - 4x + 3$ (one bend)

$x = 0 \Rightarrow y = f(0) = 3$,

so $(0,3)$ is y-intercept

$y = 0 \Rightarrow 0 = x^2 - 4x + 3$ *

$0 = (x-3)(x-1)$

$\Rightarrow x = 3$ or $x = 1$

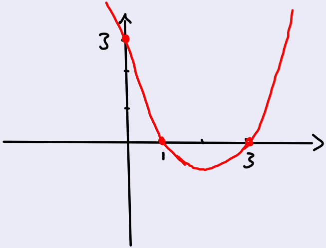
$(3,0)$, $(1,0)$ are x-intercepts

* Revise factorising ☺

Exercises

Solution

$f(x) = x^2 - 4x + 3$



Have a go at Part (iii)!
We will do this together next week ü