Lecture 1: Introduction; Numbers, Notation, Functions

MA140: Engineering Calculus.

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MA140 Course Information

Lectures

- **Lecturer:** Dr. Kirsten Pfeiffer
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  - email: kirsten.pfeiffer@universityofgalway.ie
- **When?** Tues 10am, Wed 10am, Thurs 10am

Tutorials

- Start in Week 3.
- Details will be announced later.

Other Support

SUMS ("Support for Undergraduate Mathematics and Statistics").
FREE Drop-In service on campus: also available online.
http://www.nuigalway.ie/public-sites/s-u-m-s/

MA140 Course Information

In this module, we will cover the following topics:

- Limits, Continuity, Intermediate Value Theorem
- Differentiation;
- Logarithms;
- Basic properties of integrals;
- Fundamental Theorem of Calculus;
- Methods of substitution, integration by parts, partial fractions and the Logarithm Rule.

These tools are used to tackle verbally stated engineering problems involving rates of change, maxima and minima and integration techniques.

MA140 Course Information

Assessment

- 4 Online Assignments (together worth 33 %)
- 3 Practise Tests (together worth 67 %)

Recommended Texts

- Modern Engineering Mathematics, by G. James (Prentice Hall),
- Advanced Engineering Mathematics by E. Kreyszig (Wiley), or
- Any basic Calculus textbook.

Recommended Online-Resources

- IMLSN Resources
- Paul's Online notes
- HELM Workbooks
- Check these and other online resources on the SUMS website: http://www.nuigalway.ie/public-sites/s-u-m-s/resources/

Numbers

Recall:

- $\mathbb{N} = \{1, 2, 3, 4, ...\}$ **Natural Numbers**
- $\mathbb{N}_0 = \{0, 1, 2, 3, 4, ...\}$
- $\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$ **Integers**
- $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z} \text{ and } b \neq 0\}$ **Rational Numbers**

e.g. $4 = \frac{4}{1} \in \mathbb{Q}$

$0.090909\ldots = \frac{1}{11} \in \mathbb{Q}$

$-\frac{1}{2} \in \mathbb{Q}$

$0.3333\ldots = 0.\dot{3} = 0.\bar{3} = \frac{1}{3} \in \mathbb{Q}$

What about $\pi = 3.1415926535....$?

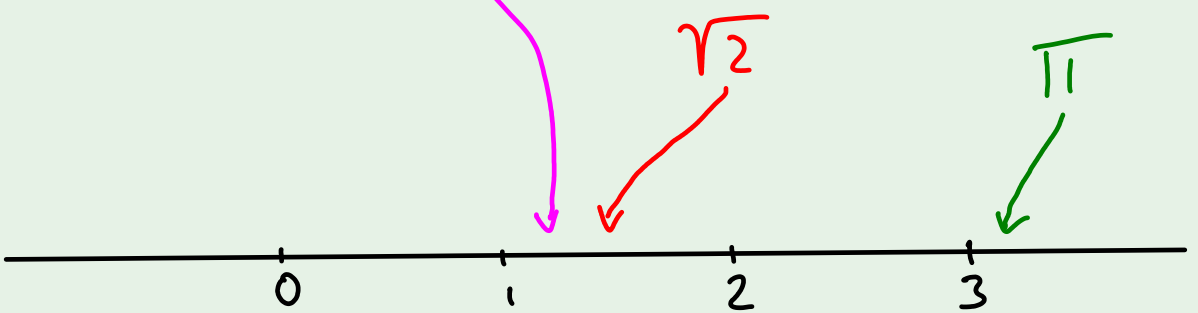
Numbers

π can not be written as a fraction, so it's **not** rational.
We say

- $\pi \notin \mathbb{Q}$,
- $\sqrt{2} \notin \mathbb{Q}$,
- $0.112374653568194.... \notin \mathbb{Q}$.

However, these numbers **do** exist on a number line.

Number Line



Numbers

A number is called **irrational** if it fits on the number line but is not rational,

e.g. $\sqrt{2}$, π

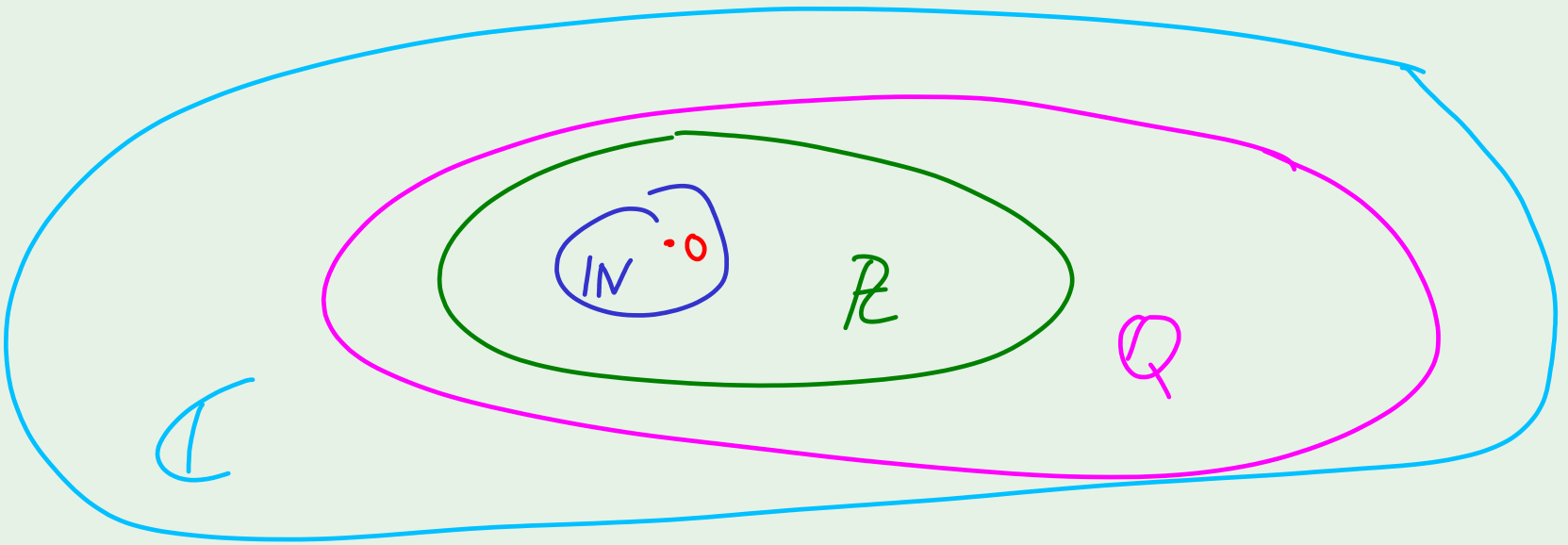
$0.01\,0\,11\,0\,111\,0\,1111\,0\ldots$ $\notin \mathbb{Q}$

- $\mathbb{R}$ is the set of **real numbers**.
  All numbers on the number line are real numbers.
- $\mathbb{C}$ is the set of **complex numbers**.
  If $c \in \mathbb{C}$ we can write

$$c = a + ib, \quad , a, b \in \mathbb{R}, \quad \mathbf{i} = \sqrt{-1}$$

Numbers

We can represent our number system visually as follows:



or write

$$\mathbb{N} \subseteq \mathbb{N}_0 \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$$

where $\subseteq$ means **“is subset of”**.

Notation

$\mathbb{R} \setminus \mathbb{Q}$ are the **irrational numbers**.

- "$\setminus$" means **"less"** or **"without"**;
- "$\in$" means **"is element of"**;
- "$\forall$" means **"for all"**;
- "$\exists$" means **"exists"**;
- "$!$" can mean **"unique"** or **"factorial"**, depending on the context.

e.g.

$$5! = (5)(4)(3)(2)(1) = 120$$

Functions

Question for you: What is a function in Mathematics?

Take a few minutes to answer the question.

Use your own words – and don't look it up on Google ;-)