Design Theory and Algebra

Warwick de Launey

IDA/CCR

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Outline

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New Book

de Launey and Flannery, Design Theory and Algebra.

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What's in this talk

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- some ideas from the book

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- some ideas from the book
- some sample results

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What's in this talk

- some ideas from the book
- some sample results
- a few words of encouragement

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Combinatorial mathematics is tremendously alive at this moment, and we believe that its greatest truths are still to be revealed.

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Important Observation

A design is a purely combinatorial concept.

Combinatorial Constraints

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A. Define the set of pairs of rows that are allowed to appear together in an array.

Orthogonality Set

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Terminology

- $\mathcal{A}$ is called the alphabet
- Λ is the orthogonality set

Pairwise Combinatorial Design

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Two rows a and b are Λ-orthogonal if

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A $\mathrm{PCD}(v, \Lambda)$ is a $v \times b$ array whose rows are pairwise Λ-orthogonal.

Error-Correcting Block Codes

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Proposition

An error-correcting code with minimum distance at least d and block-length n is a $\mathrm{PCD}(\Lambda_{\mathrm{BC}(n,d)})$.

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A Latin Square is a $\mathrm{PCD}(\Lambda_{\mathrm{LS}(n)})$.

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A Hadamard matrix is a $\mathrm{PCD}(\Lambda_{\mathrm{H}(n)})$.

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- Are there circulant $\mathrm{PCD}(\Lambda)$?

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- $v_{\max}(\Lambda_{\mathrm{BC}(n,d)})$ =??

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Theorem (de Launey and Gordon)

For all sufficiently large n, $v_{\max}(\Lambda_{\mathrm{H}(n)}) \geq \frac{1}{2}n + O(n^{113/132+o(1)})$.

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- (Graham and Shparlinski) Fairly short arithmentic sequences modulo 2^a always contain a prime power.
- So we can paste two Hadamard matrices of orders close to $n/2$ together.

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- H is a Hadamard matrix iff $H^\top$ is.
- if $H = [h_{ij}]$ is a generalized Hadamard matrix, then so is $H^* = [h^*_{ij}]$.
- if A is $(1, -1)$-matrix with all inner products equal to ± 1, then so is $A^\top$.

Group-Development for Hadamard Matrices

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Conjecture (de Launey, Horadam)

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Conjecture (Ito)

There is a relative difference set in the generalized quaternion group $\mathbf{Q}_{8t}$ *of order* $8t$ *for every* t.

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- (with Michael Smith) $a = 8$.
- (with Hadi Kharaghani) $c = 10$, $a = 4/5$.
- for any $\epsilon > 0$, we may take $a = \epsilon$ for a positive density of g's.

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Conjecture

There are enough complex complementary sequences with zero autocorrelation so that the WDL-Kharaghani construction gives a *as small as you like.*

So Where's the Algebra?

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Key Idea

Turn the arrays into matrices.

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Ambient Rings

Embed $\mathcal{A}$ into a ring $\mathcal{R}$ where the arithmetic in $\mathcal{R}$ is compatible with Λ-orthogonality.

Where We Are Heading

Modeling Λ-Equivalence

$\mathcal{R}$ contains a row group R, a column C such that two $\mathrm{PCD}(\Lambda)$ designs A and B are Λ-equivalent if and only if there exist $P \in \mathrm{Mon}(v, R)$ and $Q \in \mathrm{Mon}(v, C)$ such that

$$A = P^* B Q\,.$$

Gram Property

A is a $\mathrm{PCD}(\Lambda)$ if and only if $AA^* \in \mathrm{Gram}_{\mathcal{R}}(\Lambda)$.

The Gram Property: Involutory Rings

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1. $(r^*)^* = r$,
2. $(rs)^* = s^* r*$,
3. $(r + s)^* = r^* + s^*$.

The Gram Property II: Definition

Definition

Λ has the Gram Property over the involutory ring $\mathcal{R}$ if any $v \times v$ $(0, \mathcal{A})$-array A is a $\mathrm{PCD}(\Lambda)$ if and only if for some finite set $\mathcal{B}$,

$$AA^* = B \qquad \text{for some } B \in \mathcal{B}.$$

terminology

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terminology

1. $\mathcal{R}$ is said to have the Gram Property for Λ,
2. The set $\mathcal{B}$ is denoted $\mathrm{Gram}_{\mathcal{R}}(\Lambda)$.

Λ-Equivalence I: Two Groups

The Groups $\Pi_{\Lambda}^{\mathrm{row}}$ and $\Pi_{\Lambda}^{\mathrm{col}}$

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The Groups $\Pi_\Lambda^{\mathrm{row}}$ and $\Pi_\Lambda^{\mathrm{col}}$

1. $\Pi_\Lambda^{\mathrm{row}}$ is set of permutations ρ on $\mathcal{A}$ such that Λ is invariant under the operation where each entry $x \in \mathcal{A}$ in a row is replaced by $\rho(x)$.

Λ-Equivalence I: Two Groups

The Groups Π_Λ^{row} and Π_Λ^{col}

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2. Π_Λ^{col} is set of permutations ρ on $\mathcal{A}$ such that Λ is invariant under the operation where each entry $x \in \mathcal{A}$ in a column is replaced by $\rho(x)$.

Λ-Equivalence II: Row and Column Operations

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Definition

(row operations)

1. swaping two rows,
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Λ-Equivalence III: Definition

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Definition

Two $\mathrm{PCD}(\Lambda)$ designs are equivalent if there is a sequence of row and column operations which turns one into the other.

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Note

R and C are embeddings of $\Pi_\Lambda^{\mathrm{row}}$ and $\Pi_\Lambda^{\mathrm{col}}$ in $\mathcal{R}$.

Fundamental Existence Result

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Theorem

Every orthogonality set Λ has an ambient ring which models Λ-equivalence and has the Gram Property for Λ.

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Problem

Study these rings.

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2. Solve the matrix equation $AA^* = B$ over the ambient ring $\mathcal{R}$,
3. Solve the group ring equation $\alpha\alpha^* = b$ over the group ring $\mathcal{R}[G]$.

How to Proceed

1. Work on the hard problems,
2. Do some whimsical math,
3. Share your ideas,
4. Understand your colleagues strengths,
5. Try to find someone, who knows something that you don't know, who is willing to work with you on your problems.

IDA

My teachers (open literature):

1. K.T. Arasu (character theory),
2. David Cargo (recursions),
3. Dane Flannery (computational algebra),
4. Dan Gordon (number theory),
5. Kathy Horadam (cohomology),
6. David Levin (probability theory),
7. Martin Liebeck (representation theory),
8. Michael Smith (extraspecial p-groups, quadratic forms),
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Special thanks to my advisor: Jennifer Seberry (design theory).