# Combining Computational Algebra Systems

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University of St Andrews

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# Outline

- Motivations

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- History

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- SCIEnce

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- Future Directions

# Combining Capabilities

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  - unless they break the interface!

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- Would be nice to combine local and remote computations

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  - Supercomputers with 100 000 plus cores already exist, millions of cores are planned.
- If we want to solve bigger problems in five and ten years, we will need to exploit multiple processors

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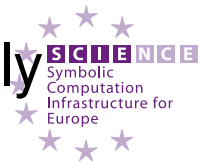
- Works OK – within fairly serious limitations



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- SAGE is essentially built around this approach

# SCIEnce: Symbolic Computation Infrastructure for Europe


**SCIEnce** – **S**ymbolic **C**omputation **I**nfrastructure in **E**urope

http://www.symbolic-computation.org

5-year **Research Infrastructure** project supported by the EU Framework VI programme grant RII3-CT-2005-026133.

# Partner institutions

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# Software Composability – one of the SCIEnce Activities

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  - do the step of my distributed computation specified by these parameters

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- Java, C and C++ libraries making it easy to interface more systems
- Supporting tools

# Three Applications

- Chosen mainly to try and convince you that it's easy to set up and use

# The application

## Some Nasty Polynomials

Let $p_1, p_2, \ldots, p_n$ be distinct prime integers. Define

$$P_{p_1,\ldots,p_n}(x) = \prod (x \pm \sqrt{p_1} \pm \sqrt{p_2} \pm \cdots \pm \sqrt{p_n})$$

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- $P_{p_1,\ldots,p_n}$ has degree $2^n$ and is irreducible over $\mathbb{Z}$ but is highly reducible over every $\mathbb{F}_p$.
- these polynomials, and others related to them are worst-case examples for standard factorisation methods.

# Setup in MUPad

```
>> package("OpenMath"):
>> swindyer:=proc(plist) <some details omitted> :
>> R := Dom::UnivariatePolynomial(x, Dom::Rational):
>> p1 := R(expand(swindyer([2,3,5,7,11]))):
>> p2 := R(expand(subs(swindyer([2,3,5,7,13,17])), x=3*x-2)):
>> p := p1 * p2:
>> degree(p), nterms(p)
96, 49
```

# Factoring

- In MuPAD

```
>> st := time(): F1 := factor(p): time()-st
38431
```

- in KANT, using a general-purpose KANT server that attempts to evaluate any OM objects sent to it

```
>> kant := SCSCP("scscp.math.tu-berlin.de", 26133):
>> st:=rtime(): F2:=kant::compute(hold(factor)(p)):rtime()
1221
```

- So calling the KANT service used only 1.2 seconds (wall-clock) instead of 38

# A Gröbner Basis Computation

## The Polynomials

$$(m_1 - a)^2 + m_2^2 - s^2, m_1^2 + (m_2 - b)^2 - s^2, (m_1 - a)^2 + (m_2 - b)^2 - s^2, -2ap_1 + 2bp_2, -2ap_2 - 2bp_1 + 4ab, aby - 1$$

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- Computing a suitable GB of these polynomials is the main part of an automated proof of the circle theorem of Appolonius

# Setup in GAP

```
gap> R := PolynomialRing(Rationals,
>           ["a","b","s","y","m1","m2","p1","p2"]);;
gap> a := R.1;; b := R.2;; s := R.3;; y := R.4;;
gap> m1 := R.5;; m2 := R.6;; p1 := R.7;; p2 := R.8;;
gap> pols := [
>     (m1-a)^2 + m2^2 - s^2,
>     m1^2 + (m2-b)^2 - s^2,
>     (m1-a)^2 + (m2-b)^2 - s^2,
>     -2*a*p1+2*b*p2,
>     -2*a*p2-2*b*p1+2*a*2*b,
>     a*b*y-1
> ];;
```

- In GAP 4.4.10, computing this GB is hard

  ```
  gap> B := GroebnerBasis(pols, MonomialGrevlexOrdering());
  ```

  does not return in a reasonable time.

# Calling a server

- A Macaulay2 server is running in Eindhoven

```
gap> I := Ideal(R, pols);;
gap> B2 := EvaluateBySCSCP("Macaulay2-Groebner", [I],
>              "scscp.win.tue.nl", 26133);;
#I  Creating a socket ...
#I  Connecting to a remote socket via TCP/IP ...
#I  Got connection initiation message
#I  Request sent ...
#I  Waiting for reply ...
gap> B2.object;
[ b-2*m2, a-2*m1, -4*m2*p2+p1^2+p2^2, m1*p1-m2*p2,
  4*m1*m2-m1*p2-m2*p1, s^2-m1^2-m2^2, y*m1*p2+y*m2*p1-1,
  4*y*m2^2*p2-p1, 4*y*m1^2*p2-4*m1+p1 ]
```

- This takes only a few seconds, mainly in the wrapper.

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- One of GAP’s extension packages `ANUPQ` provides this facility for groups of order 512
- But `ANUPQ` needs a special binary compiled and is not available for Windows, so we might wish to make this feature available as a service to call from our GAP sessions on Windows clients.

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  - Since we're only expecting GAP clients, however, we can use a GAP-specific representation – the integer given by `CodePcGroup`.
- So our server will offer just one function `IdGroup512ByCode` which will take this number and return the position of the group in the standard list (also an integer).

# Server-side Setup

```
gap> LoadPackage("scscp");; LoadPackage("anupq");;
gap> IdGroup512ByCode := function( code )
>    local G, F, H;
>    G := PcGroupCode( code, 512 );
>    F := PqStandardPresentation( G );
>    H := PcGroupFpGroup( F );
>    return IdStandardPresented512Group( H );
>    end;;
gap> InstallSCSCPprocedure("IdGroup512", IdGroup512ByCode );
InstallSCSCPprocedure : procedure IdGroup512 installed.
gap> RunSCSCPserver("localhost",26133);
```

# Client-side Wrapper

```
gap> IdGroup512:=function( G )
>    local code, result;
>    if Size( G ) <> 512 then
>      Error( "|G|<>512\n" );
>    fi;
>    code := CodePcGroup( G );
>    result := EvaluateBySCSCP("IdGroup512ByCode", [ code ],
>                              "scscp.st-and.ac.uk", 26133);
>    return result.object;
>    end;;
```

# Client-side Operation

```
gap> IdGroup512( DihedralGroup( 512 ) );
[ 512, 2042 ]
```

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  - Time for algorithm designers to start considering this.

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- Parallel versions of some library and kernel code – meataxe; random-Schreier; recognition? what else would you like?

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  - If you design interesting parallel algorithms, there should be a good way to implement them