DESIGNS, GROUPS AND COMPUTATION

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I will talk about the roles of groups and computation in the discovery, construction, classification and study of certain combinatorial designs, including new research on the topic of semi-Latin squares. Practical computational examples will be given throughout, using the DESIGN package for GAP. The lectures are intended to be accessible to a wide discrete-mathematical audience having some familiarity with group theory. It is hoped that those attending the lectures will come away with ideas on how to apply groups and the DESIGN package to design-theoretical problems which interest them.

The lectures will start with a brief overview of block designs, permutation groups and the DESIGN package. I will also give a very brief introduction to statistical optimality measures for block designs.

The lectures then move on to semi-Latin squares. Let n and k be integers, with $n > 1$ and $k > 0$. An $(n \times n)/k$ *semi-Latin square* S is an $n \times n$ array, whose entries are k-subsets of an nk-set, the set of *symbols* of S, such that each symbol of S is in exactly one entry in each row and exactly one entry in each column of S. For example, here is a $(3 \times 3)/2$ semi-Latin square:

1 4	2 5	3 6
3 5	1 6	2 4
2 6	3 4	1 5

.

Semi-Latin squares generalise Latin squares, and are used in the design of comparative experiments. I will discuss how semi-Latin squares with given properties can be constructed and classified (using the DESIGN package) via certain block designs and groups.

Of particular interest will be uniform semi-Latin squares and their existence for given parameters. A semi-Latin square is *uniform* if there is a constant μ such that any two entries of S not in the same row or column intersect in exactly μ symbols. It turns out that a uniform $(n \times n)/k$ semi-Latin square is statistically optimal (more precisely, "Schur-optimal") in the class of all $(n \times n)/k$ semi-Latin squares.

I will finish the lecture series by presenting a new, simple construction which makes an $(n \times n)/k$ semi-Latin square S from a transitive permutation group G of degree n and order nk, and discuss the relationship between certain permutation group properties of G and properties of S. In particular, if G is doubly transitive, then S is uniform, and so 2-transitive groups give us optimal semi-Latin squares with interesting combinatorial properties coming from permutation group structure.

Slides of the lecture course:

`http://www.maths.qmul.ac.uk/~leonard/debrun/lectures.pdf`

Handouts for the course:

`http://www.maths.qmul.ac.uk/~leonard/debrun/handout.txt`