



Magneto-mechanically induced wrinkling of soft magneto-active film-substrate systems

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ABSTRACT

We investigate the instability of soft magneto-active films bonded to soft semi-infinite hyperelastic substrates, with a view to revealing mechanisms that could enable on-demand surface patterning by theory and experiments. The system is subjected to a combined plane-strain mechanical load and a transverse magnetic induction field, and we use, in turn, two models for the magneto-active film, the Ideal and the Langevin magneto-elastic models. For the linearized stability analysis, we derive the Stroh formulation of the governing equations of wrinkling, and combine it with the surface impedance method to obtain exact bifurcation equations. We also derive the effective boundary conditions, approximate bifurcation equations, and finally asymptotic bifurcation equations giving the critical stretch, critical magnetic induction field, and corresponding critical wavenumbers for the onset of wrinkles. These expressions significantly improve the efficiency of the computational effort required to obtain the dispersion relations and critical fields.

We present stability bifurcation diagrams to analyze the effect of different parameters. We find that the asymptotic solution agrees well with the exact solution when the substrate-to-film shear modulus ratio r is small and the magnetic induction field is strong enough to generate a finite deformation. Then we provide the thresholds of the shear modulus ratio r_{cr}^0 or pre-stretch λ_{cr}^0 below which the film-substrate system wrinkles mechanically prior to the application of a magnetic induction field. We also investigate the influence of the particle volume fraction and find that increasing it enhances the magneto-mechanical coupling and reduces the critical magnetic induction field. Meanwhile, we find that the saturation magnetization matters significantly for strong magnetic induction fields but has no notable effect for weak fields.

Finally, we validate the theory by conducting experiments on a non-magnetic silicone elastomer substrate coated with a thin isotropic magneto-active elastic film. We also perform a simple optical reflection experiment in which magnetically induced wrinkling switches the surface response from a sharp specular spot to a broadened or fully suppressed reflection. This proof-of-concept demonstrates that the instability mechanisms uncovered here can be exploited to realize magnetically tunable optical surfaces and further, provides a fundamental basis for adaptive technologies in soft robotics, smart surfaces, smart sensors, and bioinspired systems.

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1. Introduction

Soft magneto-active (SMA) materials, such as magneto-active elastomers, represent a highly promising category of smart materials capable of reacting to magnetic field stimulation (Rigbi and Jilkén, 1983; Ginder et al., 2002; Kim et al., 2018; Mukherjee and Danas, 2022). These SMA materials can undergo significant deformation under simple, remote, and reversible actuation by magnetic fields (Bastola and Hossain, 2020; Li et al., 2025). Due to these exceptional magneto-mechanical coupling properties, SMA materials have garnered substantial academic and industrial interest, envisioning numerous potential applications, including remote actuators and sensors (Tian et al., 2011; Kim et al., 2018), soft robotics and biomedical devices (Makarova et al., 2016; Yang et al., 2023; Tang and Ren, 2024), tunable resonators and vibration absorbers (Hoang et al., 2010; Jia et al., 2024), and tunable wave devices (Yu et al., 2018; Wu et al., 2024; Alam et al., 2025; Jiang et al., 2026). Consequently, sustained academic attention has focused on formulating comprehensive nonlinear magneto-(visco)elastic theories aimed at representing the coupled magneto-mechanical response of SMA materials (Tiersten, 1964; Maugin, 1988; Dorfmann and Ogden, 2004; Destrade and Ogden, 2011; Danas et al., 2012; Saxena et al., 2013; Dorfmann and Ogden, 2014; Mukherjee and Danas, 2022). Recently, Rambašek et al. (2022) developed a general finite-strain finite-element computational framework for magnetorheological elastomers, capturing the coupled magneto-mechanical and viscoelastic responses of hard- and soft-magnetic particle systems under complex magneto-mechanical loading.

The wrinkling of film-substrate systems is a classical topic in mechanics with applications ranging from wrinkle generation in human skin to applications in material templating, assembly, and related fields (Genzer and Groenewold, 2006; Audoly and Boudaoud, 2008; Alawiye et al., 2019; Wang et al., 2023; Qiu et al., 2025). For purely elastic film-substrate systems, early efforts typically examined linearized stability about a base state undergoing large contraction (Shield et al., 1994; Cai and Fu, 2000) and were subsequently followed by more advanced investigations into wrinkling, post-buckling, semi-analytic methods, and finite element simulations (Cai and Fu, 1999; Cao and Hutchinson, 2011, 2012; Hutchinson, 2013; Fu and Ciarletta, 2015; Alawiye et al., 2019, 2020; Liu and Liu, 2026).

The coupling between Maxwell's equations and nonlinear continuum mechanics poses significant challenges for analyzing instabilities in SMA structures, including film-substrate systems. However, the pronounced and reversible magneto-mechanical coupling also enables magnetic-field-induced wrinkling as an effective strategy for programmable surface morphing and related functional applications. For example, Kirgizov et al. (2025) demonstrates that an external magnetic field can trigger the formation of microscale wrinkled protrusions on SMA films, enabling reversible surface texture reconstruction and the effective regulation of surface roughness and hydrophobic performance, as shown in Fig. 1(a). Oh et al. (2025) controlled the interaction of multimodal information through magnetic-field-induced wrinkling of SMA films embedded with arrays of hard magnetic particles, as illustrated in Fig. 1(b). Prior studies have also demonstrated magnetically reconfigurable optical surfaces and diffraction gratings using pure magneto-active elastomers. These approaches rely primarily on the microstructural reorganization of magnetic particles to alter surface roughness (Glavan et al., 2019) or utilize the magnetic shape-memory effect to temporarily emboss physical stamps into the elastomer (Lovšin et al., 2021).

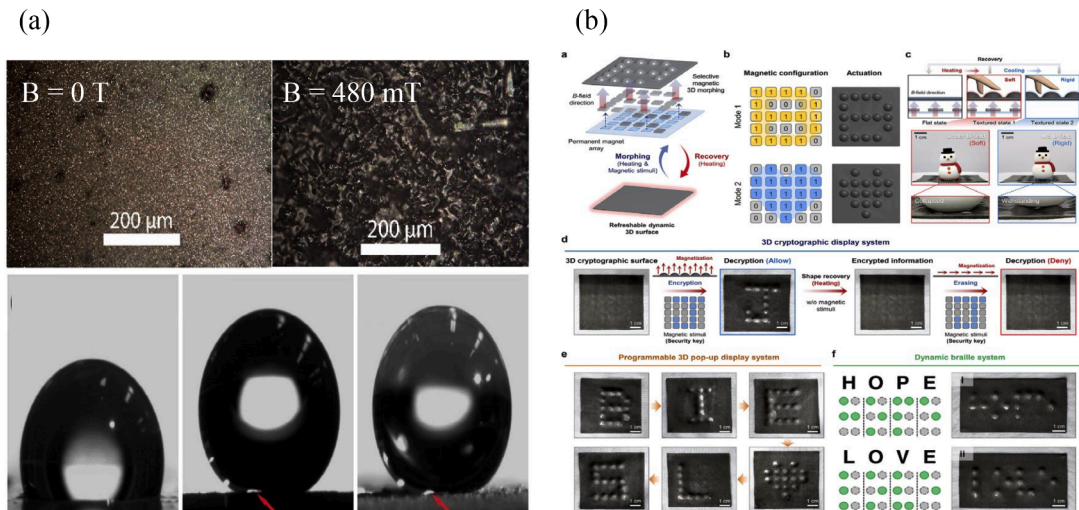


Fig. 1. Examples of magnetically induced wrinkling in SMA films enabling functional control: (a) reversible surface texture reconstruction and tunable roughness/hydrophobicity (Kirgizov et al., 2025); (b) multimodal information processing via patterned magnetic-particle-induced wrinkling (Oh et al., 2025).

Compared with standalone SMA films, SMA film-substrate systems offer enhanced design flexibility. By tailoring the geometric and mechanical properties of the film and substrate, the wrinkle characteristics and evolution behavior can be effectively controlled, thereby expanding their potential in smart surfaces and adaptive functional devices. For SMA film-substrate systems, Psarra et al. (2017, 2019) studied the stability and post-bifurcation behaviors of a nonlinear SMA film/substrate block experimentally through

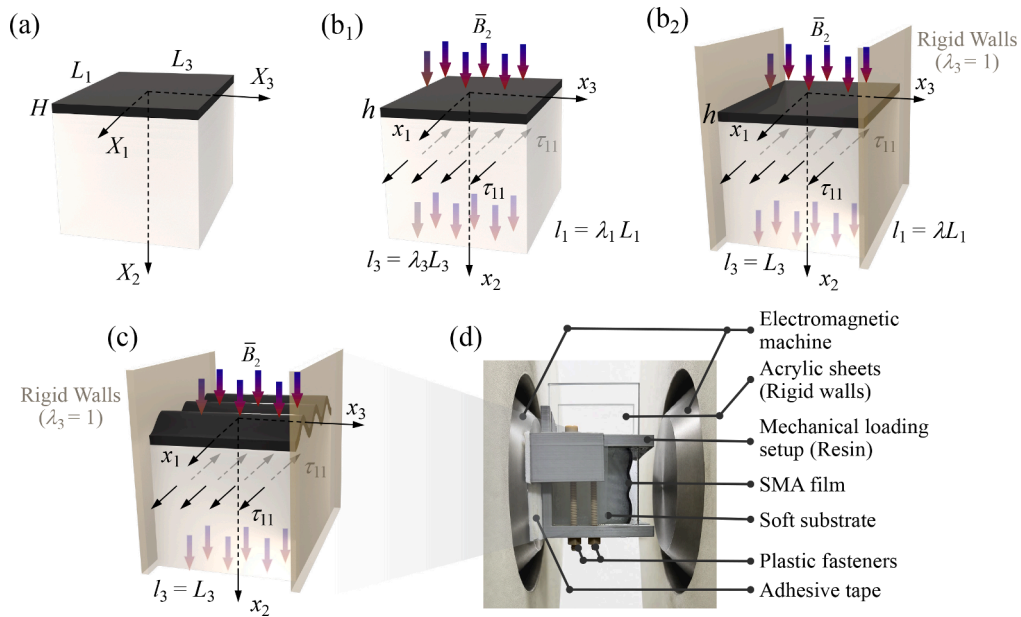


Fig. 2. Illustration of an SMA film-hyperelastic substrate system confined between two lubricated rigid walls, showing the geometric dimensions, magneto-mechanical loading conditions, and deformation states: (a) Initial undeformed configuration; (b) Deformed pre-buckling state under applied loading, including (b₁) the uniaxial case realized experimentally by Psarra et al. (2017) and (b₂) the plane-strain case with $\lambda_1 = \lambda$, $\lambda_2 = \lambda^{-1}$, and $\lambda_3 = 1$, realized experimentally in the present work; (c) Onset of surface wrinkling instability; (d) Setup for experimentally-observed wrinkling morphology under plane-strain compression.

uniaxial pre-compression experiments, demonstrating that the post-bifurcation response can be exploited to actively tune the wrinkle-induced surface roughness of an SMA film bonded to a soft non-magnetic (i.e., passive) substrate. These studies provided important experimental data and analyses on the magneto-mechanical instability and post-bifurcation behavior of SMA film-substrate systems under uniaxial compression loading. In the present work, our focus is not on a full post-bifurcation analysis or on the quantitative optimization of practical devices, but on deriving explicitly and validating experimentally the onset conditions for magnetically induced wrinkling under plane-strain loading.

From a theoretical point of view, early studies investigated the magneto-elastic buckling of beam and plate structures and later refined the underlying models to better reconcile theoretical predictions with experimental observations (Moon and Pao, 1968; Wallerstein and Peach, 1972; Miya et al., 1978; Gerbal et al., 2015). Within the nonlinear magneto-elasticity framework, some researchers have explored the surface instabilities of isotropic, finitely deformed SMA half-spaces (Otténio et al., 2007; Shahsavari and Saxena, 2025), buckling modes of rectangular SMA blocks undergoing plane-strain loading (Kankanala and Triantafyllidis, 2008), wrinkling of SMA plates subjected to plane-strain and uniaxial loading (Wu and Destrade, 2021), and macroscopic instabilities of anisotropic SMA multilayered structures (Rudykh and Bertoldi, 2013). For SMA film-substrate systems, Danas and Triantafyllidis (2014) proposed an analytical solution based on general, finite-strain, two-dimensional (2D) continuum modeling for both the SMA layer and the substrate, showing that for adequately soft substrates there is a finite-wavelength buckling mode under a transverse magnetic field. Rambašek and Danas (2021) examined the wrinkling behavior of single- and multi-layered SMA film-substrate systems under the combined action of a transverse magnetic field and in-plane biaxial pre-compression, whereas Shahsavari et al. (2026) analyzed the onset conditions for surface instability in a layered magneto-elastic half-space subjected to finite deformations and an external magnetic field. Although analytical finite-strain treatments of magneto-elastic layer/substrate bifurcation have been developed, the availability of approximate bifurcation equations and compact closed-form asymptotic onset formulae for the critical stretch, critical magnetic induction field, and critical wavenumber that directly reveal their dependence on mechanical and magnetic parameters remains limited.

The conclusion of this short literature review is that the potential instabilities and pattern formation of magneto-elastic films bonded to a soft hyperelastic substrate under large mechano-magnetic plane-strain loads (see Fig. 2) have yet to be analyzed theoretically and experimentally in detail. This work combines the advantages of SMA materials (a type of smart material) and film-substrate systems to investigate the stability of a soft magneto-elastic film bonded to a hyperelastic substrate under plane-strain loading and a uniform transverse magnetic induction field (see Fig. 2(b₂)).

Here we adopt the nonlinear magneto-elasticity theory and the associated linearized theory developed by Otténio et al. (2007) and Dorfmann and Ogden (2014) to derive the governing equations and the bifurcation equations of wrinkles for the SMA film-substrate system. To overcome the complexity of the conventional displacement-based method, we employ the Stroh formulation and the surface impedance method (Fu, 2005; Su et al., 2018, 2020; Wu and Destrade, 2021; Wu et al., 2026) to obtain explicit expressions of the bifurcation equations of the wrinkling mode (see Fig. 2(c)). We consider two magneto-elastic models (Ideal model

and Langevin model) for the SMA film, and one type of mechanical loading mode (plane-strain loading) (see Fig. 2(b₂)). We also derive the approximate bifurcation equation without matrix inversion, which improves the numerical stability of the calculation results. Finally, we present an experimental verification (see Fig. 2(d)), and its results show that the theory can accurately predict the experimental results for both plane-strain and uniaxial loading conditions.

Our results show that the onset of wrinkling in a soft film-substrate system can be actively tuned by magneto-mechanical loading, provided the material parameters are chosen appropriately. In particular, we find explicit formulas for the critical stretch and critical magnetic induction field at which wrinkles emerge, along with the corresponding critical wrinkle wavelength. We look at two loading path scenarios. In the first scenario, the applied magnetic induction field is fixed at the traction-free equilibrium value, and then an in-plane mechanical load is applied. In the second scenario, the pre-stretch λ is fixed at a given contractile or extensile level, and the magnetic induction field is increased.

The analytical bifurcation results reveal a threshold shear modulus ratio and pre-stretch (denoted r_{cr}^0 and λ_{cr}^0) below which the SMA film-substrate system wrinkles under purely mechanical compression, with no magnetic induction field applied. Unsurprisingly, this corresponds to the bifurcation criterion for the compression of non-coupled, hyperelastic systems. Above these thresholds, however, the film remains flat until a sufficient magnetic field triggers the instability, in the contractile as well as the extensile regimes.

This behavior provides a practical design guideline: by selecting the substrate-to-film shear modulus ratio and pre-stretch just above r_{cr}^0 and λ_{cr}^0 , one can ensure that the system stays smooth under mechanical load and then wrinkles *on demand* with a small applied magnetic induction field (note that if the system is pre-stretched in extension, a potentially high magnetic induction field is required for wrinkling). Comparing the Ideal model and the Langevin model, we also find that for strong magnetic induction fields, saturation magnetization is important, while for weak fields it has no significant effect, and the Ideal model suffices.

Active regulation of surface instabilities using magnetic fields represents a promising strategy in soft material research. This method is consistent with key endeavors across the development of morphing soft robotic parts, flexible and stretchable electronic devices, intelligent surface engineering, and bioinspired interface-related fields where on-demand reconfigurability is an essential requirement. Our results not only clarify the mechanisms underlying magnetically induced wrinkling but also illustrate how magnetic induction fields can be leveraged to actively regulate surface patterns and, in turn, expand the design scope for functional soft materials.

The remainder of the paper is organized as follows. In Section 2, we formulate the linearized stability problem for the SMA film/elastic substrate system, first describing the finite homogeneous deformation, then casting the incremental governing equations in Stroh form and combining them with the surface impedance method to derive the exact, approximate, and asymptotic bifurcation conditions. Section 3 presents the experimental setup and measurements and compares the theoretical predictions with the experiments. It also reports a detailed parametric study of the critical stretch, critical magnetic induction field, and corresponding critical wrinkle wavenumbers for different loading paths, shear modulus ratios, material models, and particle volume fraction. Section 4 concludes the paper by summarising the main results. There, we also put forward a proof-of-concept experiment highlighting the implications of magneto-mechanical wrinkling for magnetically tunable optical surfaces and related applications.

2. Linearized stability analysis

2.1. Finite homogeneous deformation

The system consists of an SMA film bonded to a semi-infinite hyperelastic substrate. In the initial undeformed configuration, the film and the substrate occupy the $-H < X_2 < 0$ and $X_2 > 0$ regions (see Fig. 2(a)), respectively, where (X_1, X_2, X_3) are the Cartesian coordinates, H is the film thickness, and L_1 and L_3 are the lateral lengths in the X_1 and X_3 directions, respectively. In the current deformed configuration, the film-substrate system is deformed homogeneously along the principal directions of stretch x_i (parallel to the X_i), with corresponding stretch ratios λ_i . Hence, the current film thickness is $h = \lambda_2 H$, and the lateral lengths are $\ell_1 = \lambda_1 L_1$ and $\ell_3 = \lambda_3 L_3$. The film and substrate are perfectly bonded and both are incompressible, so that they undergo the same deformations and $\lambda_1 \lambda_2 \lambda_3 = 1$.

The deformation is due to the application of mechanical forces in the x_1 and x_3 directions and of a transverse magnetic induction field, that is, the Eulerian magnetic induction vector $\mathbf{B} = [0, B_2, 0]^T$, where the superscript T signifies the transpose. The associated Lagrangian field is $\mathbf{B}_L = [0, B_{L2}, 0]^T = [0, \lambda_1 \lambda_3 B_2, 0]^T$. In general, an SMA film contracts in its plane under a transverse magnetic induction field. Here, for simplicity and to make connections with known results, we focus on the plane-strain deformation $\lambda_1 = \lambda$, $\lambda_2 = \lambda^{-1}$, $\lambda_3 = 1$, which is achieved by confining the film-substrate system between two fixed, lubricated, rigid plates, normal to the x_3 -direction, and applying external forces in the x_1 -direction, see Fig. 2(b).

To characterize the materials, we consider in turn the neo-Hookean linear magnetic model (Ideal magneto-elastic model (1)) and the neo-Hookean nonlinear magnetic model (magnetization-saturation Langevin model (2)) for the SMA film, and the neo-Hookean elastic model (3) for the soft substrate. These two magnetic models are hereafter referred to as the “Ideal model” and “Langevin model” for brevity in all subsequent discussions.

$$\Omega_f = \frac{G_f}{2} (\lambda^2 + \lambda^{-2} - 2) + \frac{1 - \chi}{2\mu_0} B_2^2, \quad (1)$$

$$\Omega_f = \frac{G_f}{2} (\lambda^2 + \lambda^{-2} - 2) + \frac{B_2^2}{2\mu_0} + \frac{\mu_0 m_f^2}{3\chi} \left\{ \ln \left(\frac{3\chi B_2}{\mu_0 m_f} \right) - \ln \left[\sinh \left(\frac{3\chi B_2}{\mu_0 m_f} \right) \right] \right\}, \quad (2)$$

$$\Omega_s = \frac{G_s}{2} (\lambda^2 + \lambda^{-2} - 2), \quad (3)$$

where G_s and G_f are the initial shear moduli of the substrate and SMA film, respectively, m_f is the saturation magnetization, $\mu_0 = 4\pi \times 10^{-7} \text{ N} \cdot \text{A}^{-2}$ is the magnetic permeability in vacuum, and χ is the relative magnetic susceptibility, related to the relative magnetic permeability μ_r through $\mu_r = \mu_f/\mu_0 = 1/(1 - \chi)$, where μ_f is the magnetic permeability of the SMA film. Note that the subscripts 's' and 'f' refer to the physical quantities of the substrate and film, respectively.

To maintain the system in the state of homogeneous deformation with stretch λ and magnetic induction field B_2 , the following dimensionless total Cauchy stress component $\bar{\sigma}_f = \sigma_f/G_f$, including the contribution of magnetic body forces, is required in the x_1 direction (Wu and Destrade, 2021),

$$\bar{\sigma}_f = \lambda^2 - \lambda^{-2} + (\chi - \frac{1}{2})\bar{B}_2^2, \quad \bar{\sigma}_f = \lambda^2 - \lambda^{-2} - \frac{1}{2}\bar{B}_2^2 + \bar{m}_f \bar{B}_2 \mathcal{L}\left(\frac{3\chi\bar{B}_2}{\bar{m}_f}\right), \quad (4)$$

in the neo-Hookean Ideal magnetic film and the Langevin film, respectively. Here $\bar{B}_2 = B_2/\sqrt{\mu_0 G_f}$ and $\bar{m}_f = m_f\sqrt{\mu_0/G_f}$ are dimensionless measures of the applied magnetic induction and saturation magnetization, respectively, and $\mathcal{L}(x) = \coth x - 1/x$ is the Langevin function. It should be noted that Eq. (4) accounts for the contribution of Maxwell stress in vacuum.

2.2. Stroh formulation and its resolution for the SMA film-substrate system

For the stability analysis, we adopt the theory of linearized incremental magneto-elasticity (Otténio et al., 2007; Dorfmann and Ogden, 2004; Destrade and Ogden, 2011) and the Stroh formalism (Su et al., 2018; Wu and Destrade, 2021). We seek 2D incremental solutions with a sinusoidal shape along the x_1 direction and amplitude variations along the x_2 direction, see Fig. 2(c), in the form

$$\{u_1, u_2, \dot{\phi}, \dot{T}_{021}, \dot{T}_{022}, \dot{B}_{L02}\} = \Re\left\{\left[k^{-1}U_1, k^{-1}U_2, \sqrt{G/\mu_0}k^{-1}\Phi, iG\Sigma_{21}, iG\Sigma_{22}, i\sqrt{G\mu_0}\Delta\right]e^{ikx_1}\right\}, \quad (5)$$

where $i = \sqrt{-1}$, U_1 , U_2 , Φ , Σ_{21} , Σ_{22} and Δ are dimensionless functions of kx_2 only, k is the wrinkling wavenumber, u_1 , u_2 are the non-zero components of the incremental displacement vector, $\dot{\phi}$ is the incremental magnetic scalar potential, $\dot{T}_{21}$, $\dot{T}_{22}$ are the non-zero components of the incremental mechanical traction vector on $x_2 = \text{constant}$ planes, and Δ is the magnitude of the incremental magnetic induction vector along the x_2 direction.

Following a standard derivation procedure (Su et al., 2018, 2020; Wu and Destrade, 2021), the dimensionless incremental equations of equilibrium can be formulated as a first-order differential equation, $\eta' = i\mathbf{N}\eta$, where $\eta = [U_1 \ U_2 \ \Phi \ \Sigma_{21} \ \Sigma_{22} \ \Delta]^T$ is the generalised displacement-traction Stroh vector, $\mathbf{N}$ is the constant 6×6 Stroh matrix, and the prime denotes differentiation with respect to kx_2 . With solutions in the form $\eta(kx_2) = \eta^0 e^{iqkx_2}$, the resulting eigenvalue problem $(\mathbf{N} - q\mathbf{I})\eta^0 = \mathbf{0}$ provides three pairs of complex conjugate eigenvalues q_j and six eigenvectors $\eta^{(j)}$ in the SMA film ($-h \leq x_2 \leq 0$), with the general solution as

$$\eta(kx_2) = \begin{bmatrix} \mathbf{U}(kx_2) \\ \mathbf{S}(kx_2) \end{bmatrix} = \sum_{j=1}^6 c_j \eta^{(j)} e^{iq_j kx_2}, \quad (6)$$

where $\mathbf{U} = [U_1 \ U_2 \ \Phi]^T$ and $\mathbf{S} = [\Sigma_{21} \ \Sigma_{22} \ \Delta]^T$ are the generalised displacement and traction vectors, respectively, and c_j ($j = 1, \dots, 6$) are arbitrary constants to be determined.

For the neo-Hookean Ideal magneto-elastic model (1) under plane-strain loading, we find the 3×3 sub-matrices of $\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 & \mathbf{N}_2 \\ \mathbf{N}_3 & \mathbf{N}_1^T \end{bmatrix}$ as

$$\mathbf{N}_1 = \begin{bmatrix} 0 & \bar{B}_2^2 \lambda^2/2 - 1 & \bar{B}_2 \lambda^2 \\ -1 & 0 & 0 \\ 2\bar{B}_2(1 - \chi) & 0 & 0 \end{bmatrix}, \quad \mathbf{N}_2 = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \chi - 1 \end{bmatrix}, \quad (7)$$

$$\mathbf{N}_3 = \begin{bmatrix} -3\lambda^{-2} - \lambda^2 - \bar{B}_2^2(2 - 3\chi) & 0 & 0 \\ 0 & \bar{B}_2^2(1 - \chi) - \lambda^2 + (\lambda^{-1} - \bar{B}_2^2 \lambda/2)^2 & \bar{B}_2^3 \lambda^2/2 \\ 0 & \bar{B}_2^3 \lambda^2/2 & (1 - \chi)^{-1} + \lambda^2 \bar{B}_2^2 \end{bmatrix}. \quad (8)$$

The first three eigenvalues are $q_1 = i$, $q_2 = i\lambda^2$, $q_3 = i$, with the associated eigenvectors

$$\begin{aligned} \eta^{(1)} &= [1, i, -i\bar{B}_2(1 - \chi), i(2\lambda^{-2} + \bar{B}_2^2(1 - 2\chi)/2), \bar{B}_2^2/2 - \lambda^2 - \lambda^{-2}, \bar{B}_2]^T, \\ \eta^{(2)} &= [\lambda^2, i, -i\bar{B}_2(1 - \chi), i(\lambda^2 + \lambda^{-2} + \bar{B}_2^2(1 - 2\chi)/2), \bar{B}_2^2 \lambda^2/2 - 2, \bar{B}_2 \lambda^2]^T, \\ \eta^{(3)} &= [0, 0, i, -i\bar{B}_2, \bar{B}_2, (1 - \chi)^{-1}]^T, \end{aligned} \quad (9)$$

and the last three eigenvalues and eigenvectors are their respective complex conjugates.

Note that we also find explicit expressions for these quantities in the case of the neo-Hookean magnetization saturation Langevin model (2), but they are lengthy and are relegated to Supplementary Material A (SM-A).

2.3. Bifurcation equations for wrinkling instabilities

We may build a general solution to the incremental equations of equilibrium as linear combinations of the eigensolutions in the SMA film and substrate, and then apply the incremental boundary conditions to yield a homogeneous system for the arbitrary constants. The Determinantal Method consists in writing the solvability of this system. However, it is ill-posed and often runs into numerical stiffness issues. In our case, the incremental boundary conditions (Otténio et al., 2007; Wu and Destrade, 2021) are quite intricate because, in addition to the continuity of the Stroh vectors at the film-substrate interface, we also have to account for the Maxwell stress in the $x_2 \leq -h$ (vacuum) and $x_2 \geq 0$ (hyperelastic substrate) regions. For these reasons, we turn to the Surface Impedance Matrix Method, which is numerically robust and yields exact and asymptotic analytical expressions.

2.3.1. Exact solution

We relate the Stroh vectors at the faces of the SMA film through $\eta(-kh) = \mathbf{M}\eta(0)$, which can be derived from Eq. (6), where $\mathbf{M} = \mathcal{N}^T \mathbf{E} \mathcal{N}^{-1}$, with $\mathcal{N} = [\eta^{(1)} \dots \eta^{(6)}]$ being the matrix of eigenvectors and $\mathbf{E} = \text{diag}[e^{-iq_1 kh}, \dots, e^{-iq_6 kh}]$. The $\mathbf{M}$ matrix has the following block structure of 3×3 submatrices (Shuvalov, 2000),

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_3 & \mathbf{M}_1^T \end{bmatrix}, \quad (10)$$

and can be used to relate the generalised face tractions to the generalised face displacements through the film impedance matrix $\mathbf{Z}_f$ as follows,

$$\begin{bmatrix} -\mathbf{S}(0) \\ \mathbf{S}(-kh) \end{bmatrix} = i\mathbf{Z}_f \begin{bmatrix} \mathbf{U}(0) \\ \mathbf{U}(-kh) \end{bmatrix}, \quad \mathbf{Z}_f = i \begin{bmatrix} -\mathbf{M}_2^{-1}\mathbf{M}_1 & \mathbf{M}_2^{-1} \\ -\mathbf{M}_3 + \mathbf{M}_1^T\mathbf{M}_2^{-1}\mathbf{M}_1 & -\mathbf{M}_1^T\mathbf{M}_2^{-1} \end{bmatrix}. \quad (11)$$

Then, recalling that the SMA film is sandwiched between two half-spaces where the Maxwell stress is present, it can be concluded that the exact bifurcation condition reads (see Shuvalov and Every 2002 for the purely elastic counterpart)

$$\det \left[\mathbf{Z}_f + \begin{pmatrix} r\mathbf{z}_s + \mathbf{z}^* & \mathbf{0} \\ \mathbf{0} & \mathbf{z}^{*T} \end{pmatrix} \right] = 0, \quad (12)$$

where $r = G_s/G_f$ is the substrate-to-film shear modulus ratio, and

$$\mathbf{z}_s = \begin{bmatrix} \lambda^{-2} + 1 & i(1 - \lambda^{-2}) & 0 \\ -i(1 - \lambda^{-2}) & \lambda^2 + 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{z}^* = \begin{bmatrix} 0 & i\bar{B}_2^2/2 & i\bar{B}_2 \\ -i\bar{B}_2^2/2 & -\bar{B}_2^2 & -\bar{B}_2 \\ -i\bar{B}_2 & -\bar{B}_2 & -1 \end{bmatrix}, \quad (13)$$

are the surface impedance matrices for the neo-Hookean elastic half-space in the absence of magnetic induction fields and for the vacuum half-space in the presence of the Maxwell stress, respectively. A detailed derivation of $\mathbf{z}^*$ is given in Wu and Destrade (2021), and that of Eq. (12) is provided in SM-B1.

Using block algebra, we can transform the bifurcation Eq. (12) into the following equivalent form,

$$\det \left\{ \mathbf{z}^{*T} - i[\mathbf{M}_3 + i\mathbf{M}_1^T(r\mathbf{z}_s + \mathbf{z}^*)][\mathbf{M}_1 + i\mathbf{M}_2(r\mathbf{z}_s + \mathbf{z}^*)]^{-1} \right\} = 0. \quad (14)$$

The detailed derivation of Eq. (14) is presented in SM-B2, and SM-B3 further demonstrates its mathematical equivalence to Eq. (12).

We can then plot the bifurcation (or dispersion) curves for a given shear modulus ratio r : either the $\lambda - kH$ curves when we consider wrinkling instability under an increasing mechanical compression for a fixed magnetic induction field, or the $\bar{B}_2 - kH$ curves when we focus on an increasing magnetic induction field for a fixed pre-stretch. Typically, these curves exhibit an extremum: a maximum $\lambda = \lambda_{cr}$ in the former case, a minimum $\bar{B}_2 = \bar{B}_{cr}$ in the latter case (although not always), see Section 3. These extrema are the sought critical stretches and critical magnetic induction fields of primary wrinkling bifurcation.

2.3.2. Approximate solution

In Section 3, we show that, numerically, the critical fields occur in the early part of the bifurcation curves, when kh is small. It thus makes sense to seek an approximate bifurcation condition of the exact one (14) using a power series expansion. We expand the relationship $\eta(-kh) = \mathbf{M}\eta(0)$ up to the sixth power in kh , apply the incremental boundary conditions, and observe that the resulting approximate bifurcation equation captures accurately the extrema corresponding to the critical wrinkling values (see Section 3).

Earlier we wrote that integrating $\eta' = i\mathbf{N}\eta$ from 0 to $-kh$ gave $\eta(-kh) = \mathbf{M}\eta(0)$, where $\mathbf{M} = \exp(-ikh\mathbf{N})$. For small kh , this matrix can be expanded into a power series as $\mathbf{M} = \sum \frac{(-ikh)^n}{n!} \mathbf{N}^n$. Using the incremental magneto-mechanical boundary conditions on the upper and lower surfaces of the SMA film and truncating the power-series expansion at the m -th order, we obtain the effective boundary conditions as

$$\left\{ \sum_{n=0}^m \frac{(-ikh)^n}{n!} [\mathbf{K}_{n3} + (i\mathbf{K}_{n1}^T - \mathbf{z}^{*T}\mathbf{K}_{n2})\mathbf{z}^* + i\mathbf{z}^{*T}\mathbf{K}_{n1}] \right\} \mathbf{U}_s(0) = - \left[\sum_{n=0}^m \frac{(-ikh)^n}{n!} (\mathbf{K}_{n1}^T + i\mathbf{z}^{*T}\mathbf{K}_{n2}) \right] r\mathbf{S}_s(0), \quad (15)$$

where $\mathbf{K}_n \equiv \mathbf{N}^n$ has the following block structure $\mathbf{K}_n = \begin{bmatrix} \mathbf{K}_{n1} & \mathbf{K}_{n2} \\ \mathbf{K}_{n3} & \mathbf{K}_{n1}^T \end{bmatrix}$, and $\mathbf{K}_{n2}, \mathbf{K}_{n3}$ are symmetric (Destrade, 2003). Note that $\mathbf{K}_{01} = \mathbf{K}_{04} = \mathbf{I}_{3 \times 3}$ and $\mathbf{K}_{02} = \mathbf{K}_{03} = \mathbf{0}$.

Eq. (15) can be recast into the form $\mathbf{S}_s(0) = \mathbf{z}_{\text{eff}} \mathbf{U}_s(0)$, where $\mathbf{z}_{\text{eff}}$ is an effective boundary operator (Wu et al., 2015), in the sense that, from the substrate viewpoint, the SMA film and the vacuum half-space have been replaced by the modified boundary condition (15). Combining Eq. (15) with the surface impedance relation $\mathbf{S}_s(0) = i\mathbf{z}_s \mathbf{U}_s(0)$ of the hyperelastic substrate, the approximate bifurcation equation at the m -th order is obtained as

$$\det \left\{ \sum_{n=0}^m \frac{(-ikh)^n}{n!} [\mathbf{z}^{*T} \mathbf{K}_{n1} + (\mathbf{K}_{n1}^T + i\mathbf{z}^{*T} \mathbf{K}_{n2})(r\mathbf{z}_s + \mathbf{z}^*) - i\mathbf{K}_{n3}] \right\} = 0. \quad (16)$$

The detailed derivations of the effective boundary condition (15) and the approximate bifurcation Eq. (16) are provided in SM-C. Alternatively, multiplying the bifurcation condition (14) by $\det [\mathbf{M}_1 + i\mathbf{M}_2(r\mathbf{z}_s + \mathbf{z}^*)]$ and truncating the series at the m -th order yields Eq. (16) as well. Compared with the direct algebraic truncation approach, the effective boundary condition provides a physically transparent and readily extendable framework for magneto-mechanical wrinkling analysis.

Solving the approximate bifurcation Eq. (16) numerically for $m = 6$ is more efficient and less computationally expensive than solving the exact bifurcation condition (12) or (14), and it is highly accurate as we show in Section 3.5.

2.3.3. Asymptotic solutions for small substrate-to-film shear modulus ratios

The numerical root-finding method for the exact bifurcation Eq. (14) leads to substantial computational expense, especially when it comes to determining the extrema. In contrast, asymptotic expansions offer an efficient alternative for determining the critical values of stretch, magnetic induction field, and corresponding critical wavenumber. In this section, we specialise the analysis to the neo-Hookean Ideal magneto-elastic model (1) for the SMA film and the neo-Hookean elastic model (3) for the soft substrate.

Assuming $kh \ll 1$, expanding the approximate bifurcation Eq. (16) leads to an approximate bifurcation condition expressed as a sixth-order power series in kh (see Eq. (D1) in SM-D). With the assumption that $r = \mathcal{O}((kh)^2)$ in the case where the SMA film is much stiffer than the substrate, and neglecting terms of order $(kh)^4$ and higher, we derive asymptotic expression for the stretch λ under a fixed magnetic induction field $\bar{B}_2$ as follows (see SM-D1),

$$\lambda = \lambda_0 + \omega_0(kh) + \omega_1\left(\frac{r}{kh}\right) + \omega_2(kh)^2 + \omega_3r + \omega_4\left(\frac{r}{kh}\right)^2 + \omega_5(kh)^3 + \omega_6r(kh) + \omega_7\frac{r^2}{kh} + \omega_8\left(\frac{r}{kh}\right)^3, \quad (17)$$

where

$$\lambda_0 = \sqrt{\frac{\bar{B}_2^2 \chi^2 + \sqrt{\bar{B}_2^4 \chi^4 + 4(1 - \chi)^2}}{2(1 - \chi)}} \geq 1 \quad (18)$$

is the critical stretch in the thin-plate limit (Wu and Destrade, 2021), and the coefficients are

$$\omega_0 = -\frac{\lambda_0(\lambda_0^4 - 1)}{4(1 - \chi)(\lambda_0^4 + 1)} \leq 0, \quad \omega_1 = -\frac{\lambda_0^3(\lambda_0^2 + 1)}{2(\lambda_0^4 + 1)} < 0, \quad (19)$$

while $\omega_2, \dots, \omega_8$ are given explicitly in SM-D1, see Eq. (D9).

Then, by differentiating Eq. (17) with respect to kh and setting the first-order derivative equal to zero, we find where the stretch is maximized along the bifurcation curve, and obtain the following asymptotic expansions for the critical stretch up to order r (neglecting terms of order $r^{3/2}$ and higher)

$$\lambda_{\text{cr}} = \lambda_0 + 2(\omega_0\omega_1r)^{1/2} + \left(\omega_3 + \frac{\omega_0\omega_4}{\omega_1} + \frac{\omega_1\omega_2}{\omega_0}\right)r, \quad (20)$$

and for the critical wavenumber up to order r

$$(kh)_{\text{cr}} = \left(\frac{\omega_1}{\omega_0}r\right)^{1/2} + \left(\frac{\omega_4}{\omega_1} - \frac{\omega_1\omega_2}{\omega_0^2}\right)r. \quad (21)$$

When the SMA film is not magneto-active, we take $\bar{B}_2 = 0$, $\chi = 0$, and find $\lambda_0 = 1$ as expected. Moreover, we then recover the asymptotic formula of Cai and Fu (2000) for the bifurcation of an elastic film-substrate system when $r = r_0(kh)^2$ for some zeroth-order constant r_0 . Taking the incompressible limit in their equation (31), we find $\lambda = 1 - (r_0/2)kh$, which corresponds to the starting terms of Eq. (17) in the non-magnetic specialisations of Eq. (19), i.e. $\omega_0 = 0$ and $\omega_1 = -1/2$.

Similarly, we obtain the asymptotic expansion of the squared dimensionless magnetic induction field for a fixed pre-stretch $\lambda \geq 1$ in the form

$$\bar{B}_2^2 = \bar{B}_0^2 + \kappa_0(kh) + \kappa_1\left(\frac{r}{kh}\right) + \kappa_2(kh)^2 + \kappa_3r + \kappa_4\left(\frac{r}{kh}\right)^2 + \kappa_5(kh)^3 + \kappa_6r(kh) + \kappa_7\frac{r^2}{kh} + \kappa_8\left(\frac{r}{kh}\right)^3, \quad (22)$$

where

$$\bar{B}_0^2 = \frac{(\lambda^4 - 1)(1 - \chi)}{\lambda^2 \chi^2}, \quad (23)$$

and the coefficients are

$$\kappa_0 = \frac{\lambda^4 - 1}{2\chi^2 \lambda^2}, \quad \kappa_1 = \frac{(1 - \chi)(1 + \lambda^2)}{\chi^2}, \quad \kappa_2 = \frac{(1 - \chi)\lambda^2}{3\chi^2}, \quad (24)$$

Table 1

Geometric, magnetic, and material parameters used for the SMA film ($i = f$) and soft substrate ($i = s$). The three parameters (H_i , c , χ) are measured experimentally, whereas the other two (G_i , $\mu_0 m_i$) are taken from Psarra et al. (2017, 2019).

Parameters	H_i (mm)	c (-)	G_i (kPa)	χ (-)	$\mu_0 m_i$ (T)
SMA film	0.8	0.2	10	0.4	0.5
Hyperelastic substrate	39.2	0.0	3	0.0	0.0

while $\kappa_3, \dots, \kappa_8$ are given explicitly in SM-D2, see Eq. (D14).

Again, setting the derivative of Eq. (22) with respect to kh to zero determines the location of the (local) minimum. Neglecting terms of order $r^{3/2}$ and higher, we obtain the squared critical magnetic induction field $\bar{B}_{cr}^2$ as

$$\bar{B}_{cr}^2 = \bar{B}_0^2 + 2(\kappa_0 \kappa_1 r)^{1/2} + \left(\kappa_3 + \frac{\kappa_0 \kappa_4}{\kappa_1} + \frac{\kappa_1 \kappa_2}{\kappa_0} \right) r, \quad (25)$$

and the corresponding critical wavenumber is

$$(kh)^{cr} = \left(\frac{\kappa_1}{\kappa_0} r \right)^{1/2} + \left(\frac{\kappa_4}{\kappa_1} - \frac{\kappa_1 \kappa_2}{\kappa_0^2} \right) r. \quad (26)$$

Note that in SM-D we present the asymptotic expansions of λ_{cr} and $\bar{B}_{cr}^2$ up to order $r^{3/2}$. Note also that these expansions, based on the assumption that r is of order $(kh)^2$, are only valid when the effect of the magneto-elastic coupling is strong enough to deform the coated half-space. If $\bar{B}_0$ is weak, then a different assumption is required, which is why the asymptotic expansions do not recover the classic hyperelastic results of Fu and coworkers (Cai and Fu, 1999, 2000) as $\bar{B}_0 \rightarrow 0$, because they are based on the assumption that r is of order $(kh)^3$.

3. Results and discussion

3.1. Experiments

For our experiments, we designed a setup similar to that reported in Psarra et al. (2017). The objectives were to employ the proposed theory to validate their wrinkling results under uniaxial compression, extend the investigation to the plane-strain compression case, and further assess the predictive capability of our theoretical framework. Hence, our experimental film-substrate system consisted of an elastomeric cube with side length 40 mm, composed of an isotropic magneto-sensitive elastic film of thickness $H_f = 0.8$ mm bonded onto a soft non-magnetic substrate of thickness $H_s = 39.2$ mm. The substrate was fabricated from silicone elastomer Ecoflex 00–10 (Smooth-On, Inc., Macungie, PA 18062, USA), mixed in a 1:1 wt ratio of polymer to hardener. The SMA film was fabricated from the stiffer silicone elastomer Ecoflex 00–50, uniformly mixed with 5 μm -diameter carbonyl iron particles.

We performed uniaxial tensile tests on the standard dogbone samples of the SMA materials, and confirmed that the shear modulus increases with increasing iron-particle volume fraction c . However, as noted by Psarra et al. (2017, 2019), the shear moduli measured by uniaxial tensile tests cannot be used directly for the wrinkling experiments of the film-substrate systems, and should serve only as a guide because the effective mechanical response relevant to wrinkling depends on the actual film-substrate specimen and loading conditions, including the curing/polymerization process of the full block, the pre-compressed and relaxed state in the fixed setup, and the rate-dependent nature of tensile testing. Instead, we first adopted the same shear moduli as those reported by Psarra et al. (2017, 2019) for the SMA film with particle volume fraction $c = 0.2$ in their experiments, as listed in Table 1. Specifically, Psarra et al. (2017) identified the shear moduli of the substrate and the SMA film by fitting experimentally measured bifurcation curves with full-field finite-element simulations.

The SMA film was prepared using the spin-coating method (Meyerhofer, 1978). First, we poured the mixture into a $40 \times 40 \times 0.8$ mm³ mold base. The mold was placed at the center of a Rubber sheeting machine (Tianjin Jinlin Co. LTD, KW-4B) and rotated at 200 rpm, so that the mixture was evenly dispersed in the mold and the excess part was thrown out. Then, to build the whole $40 \times 40 \times 40$ mm³ system, another mold was mounted onto the aforementioned base and the substrate mixture was poured on top of the film. Finally, after vacuuming, the whole structure was placed in an oven kept at 70 °C for two hours. After demolding, the loading experiments were conducted using an electromagnetic machine (Litian Magetoelastic Technology Co. LTD, PEM 800-5). Fig. 3 shows a schematic illustration of the fabrication and experimental procedure.

We used a SQUID-VSM magnetometer to measure the SMA film's permeability with different iron-particle volume fractions c , with the aim of determining the appropriate volume fraction of magnetic particles for the subsequent experiments. We found that the experimental results are in good agreement with the predictions of the Maxwell-Garnett model (Psarra et al., 2017), $\chi = 3c/(1 + 2c)$, at least for small values of c , as can be observed in Fig. 4(a). When the volume fraction of magnetic particles exceeds a certain threshold ($c \approx 0.3$), the experimental results deviate from the predicted values, as inter-particle interactions become non-negligible. Therefore, the subsequent analysis is restricted to the range of $c \leq 0.3$. Previous experimental studies on related materials (Danas et al., 2012) indicate that pure carbonyl iron has a saturation magnetization of approximately $\mu_0 m_{\text{iron}} = 2.5$ T. For the SMA film

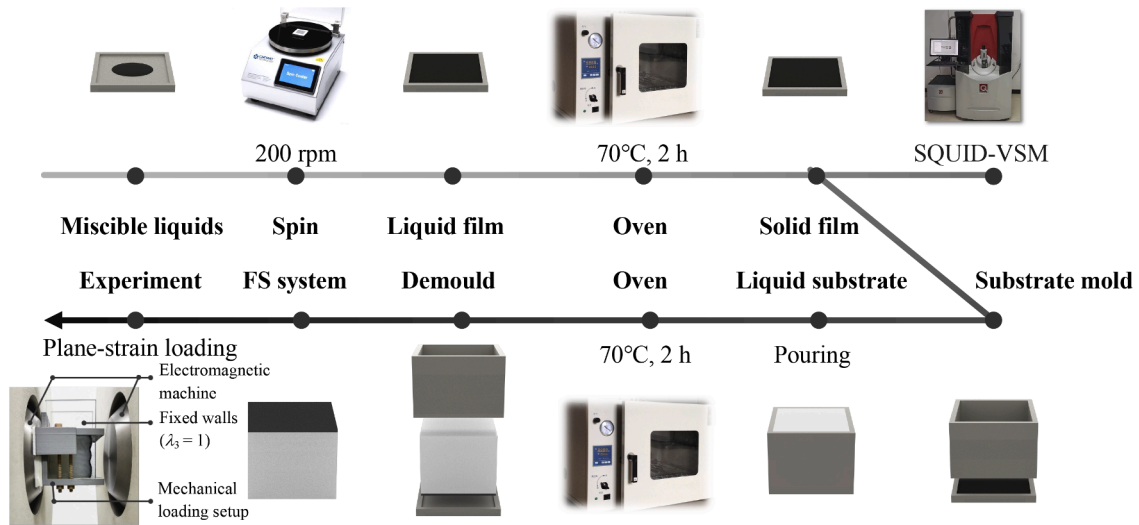


Fig. 3. Overview of the fabrication procedure and experimental setup for surface wrinkling experiments of the SMA film-hyperelastic substrate system.

containing a carbonyl-iron volume fraction of $c = 0.2$ uniformly dispersed within a non-magnetic polymer matrix, $\chi = 0.4$ and the corresponding saturation magnetization is estimated as $\mu_0 m_f = 0.5$ T, as reported in Psarra et al. (2017, 2019) and listed in Table 1.

For the wrinkling experiments of the SMA film-substrate system, we selected the pre-stretches $\lambda = 0.85, 0.875, 0.9, 0.925, 0.95$ in the plane-strain case. To enforce the plane-strain compression, we first used a compressor to fix the stretch λ , and then put the system under a transverse magnetic load (see Figs. 2(d) and 3). We also report the experimental results of Psarra et al. (2017, 2019), who performed uniaxial pre-compression tests at $\lambda = 0.85, 0.88, 0.9, 0.95$. There, we determined the approximate critical magnetic induction field corresponding to the occurrence of wrinkles by analyzing their bifurcation amplitude curves. We supplemented this data with our own additional experimental results at $\lambda = 0.925$.

Note that the experimental substrate thickness (39.2 mm) is much larger (almost 50 times) than the SMA film thickness (0.8 mm), and the critical wavelength is far smaller than the substrate thickness, so that boundary effects from the bottom surface of the substrate are expected to be negligible near the onset of wrinkling. It follows that our theoretical treatment of the substrate as a half-space is justified. Lateral friction forces at the lubricated rigid walls may nevertheless affect the measured critical magnetic induction field, particularly at large compression, and lead to a major source of experimental and theoretical errors (Psarra et al., 2019). Accordingly, we did not consider the cases where the pre-stretch is small ($\lambda < 0.85$).

Under the uniaxial and plane-strain states, the comparisons between experiment and theory (for the neo-Hookean Ideal and Langevin models) are shown in Fig. 4(b). Table 2 displays the values of the experimental critical magnetic induction field $\bar{B}_{cr}^{exp}$ and the theoretical results $\bar{B}_{cr}$ of the neo-Hookean Ideal magneto-elastic model. For our plane-strain case, six repeated tests were conducted at each prescribed stretch. The reported value is the mean of the six extracted critical magnetic induction fields, and the error bar represents the sample standard deviation (SD). The specific SD values are listed in Table 2.

For the uniaxial data from Psarra et al. (2017, 2019), except for $\lambda = 0.925$ (our data), the uncertainty does not represent the SD of repeated tests performed in the present work. Instead, it represents the uncertainty associated with extracting the onset field from the published bifurcation amplitude curves (Fig. 5(a) in Psarra et al. (2017)). In practice, we identified a small interval around the zero-amplitude branch: the lower bound corresponds to the magnetic induction field for which the amplitude remains indistinguishable from zero, and the upper bound corresponds to the magnetic induction field for which a finite amplitude is observed. The critical magnetic induction field $\bar{B}_{cr}^{exp}$ was taken as the midpoint of this interval. Thus, the error bars in Fig. 4(b) represent the SD values for the present plane-strain measurements and the extraction uncertainties for the uniaxial data adopted from Psarra et al. (2017, 2019).

We see that the experimental results are in good agreement with the theoretical predictions: the relative errors are below 3.6% in both the plane-strain and uniaxial cases. An exception occurs at $\lambda = 0.95$, where the errors increase to about 8.71% and 4.34% for the plane-strain and uniaxial cases, respectively, because the experiments show that the wrinkling pattern becomes 2D, departing from the one-dimensional (1D) pattern studied here. This indicates a limitation of the present onset analysis: it does not capture mode competition between 1D and 2D patterns, localization, or post-bifurcation pattern transitions. These effects may become particularly relevant in regimes where functional surface responses depend on the developed morphology rather than only on the wrinkling onset threshold.

3.2. Nonlinear magnetization response

As a preliminary step, we first examine the nonlinear static magnetization response of the SMA film-substrate system before analyzing the wrinkling bifurcation behavior. Following Wu and Destrade (2021), the governing equations for the magnetic response

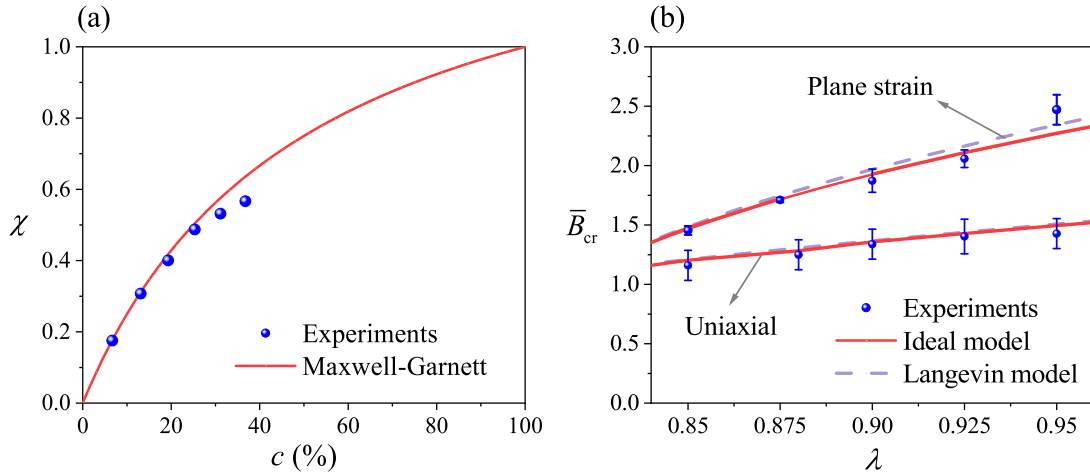


Fig. 4. (a) Experimental measurements and analytical estimates of the SMA film's relative magnetic susceptibility χ for different iron-particle volume fractions c ; (b) Comparison between theoretical predictions and experimental measurements under plane-strain and uniaxial conditions, where the plane-strain experimental results are obtained in the present study, whereas the uniaxial data are adopted from Psarra et al. (2017, 2019), except for $\lambda = 0.925$, which is measured in the present study.

Table 2

Dimensionless critical magnetic induction field under different pre-stretch levels λ and loading conditions (plane-strain and uniaxial) for a shear modulus ratio of $r = G_s/G_f = 0.3$. The superscript “exp” denotes the experimental results.

	Value for $\lambda = 0.85$	Value for $\lambda = 0.875$	Value for $\lambda = 0.9$	Value for $\lambda = 0.925$	Value for $\lambda = 0.95$
Critical values for the plane-strain case					
$\bar{B}_{cr}$	1.47583	1.72122	1.92843	2.10975	2.2722
$\bar{B}_{cr}^{exp}$	1.45	1.71	1.87	2.06	2.47
Error (%)	1.75	0.65	3.03	2.36	8.71
SD	0.03785	0.02082	0.09777	0.07380	0.12616
	Value for $\lambda = 0.85$	Value for $\lambda = 0.88$	Value for $\lambda = 0.9$	Value for $\lambda = 0.925$	Value for $\lambda = 0.95$
Critical values for the uniaxial case					
$\bar{B}_{cr}$	1.20313	1.28289	1.35755	1.42797	1.49485
$\bar{B}_{cr}^{exp}$	1.16	1.25	1.34	1.40	1.43
Error (%)	3.58	2.56	1.29	1.96	4.34

are

$$M_2 = \mu_0^{-1} \chi B_2, \quad (27)$$

for the Ideal model, and

$$M_2 = m_f \frac{B_2}{|B_2|} \mathcal{L}\left(\frac{3\chi|B_2|}{\mu_0 m_f}\right), \quad (28)$$

for the saturation Langevin model. In the limit of a weak magnetic induction field ($B_2 \rightarrow 0$), the nonlinear saturation Langevin model recovers the linear magnetization relation of the Ideal model.

Fig. 5(a) presents the magnetization response curves, i.e., the dimensionless magnetization $\bar{M}_2^* = \bar{M}_2/\bar{m}_f$ as a function of the dimensionless magnetic induction field $\bar{B}_2^* = \bar{B}_2/\bar{m}_f$, for the Ideal model (solid lines) and the Langevin model (dashed lines) with different particle volume fractions ($c = 0.1, 0.2$, and 0.3). Here, $\bar{m}_f = m_f \sqrt{\mu_0/G_f}$, $\bar{M}_2 = M_2 \sqrt{\mu_0/G_f}$, and μ_0 and G_f denote the vacuum magnetic permeability and the initial shear modulus of the SMA film, respectively.

To quantitatively characterize the deviation between the two constitutive models, Fig. 5(b) further shows the relative error δ of the dimensionless magnetization $\bar{M}_2^*$ predicted by the Langevin model with respect to that given by the Ideal model. The relative magnetic susceptibility is calculated as $\chi = 3c/(1 + 2c)$. The corresponding saturation magnetization is estimated as $\mu_0 m_f = c \mu_0 m_{\text{iron}} = 2.5c$ T.

As shown in Fig. 5(a), the neo-Hookean Ideal model exhibits a linear magnetization response, whereas the magnetization response of the SMA film-substrate system described by the saturation Langevin model becomes nonlinear due to magnetic saturation. For particle volume fractions of $c = 0.1, 0.2$, and 0.3 , respectively, the magnetization responses predicted by the two material models

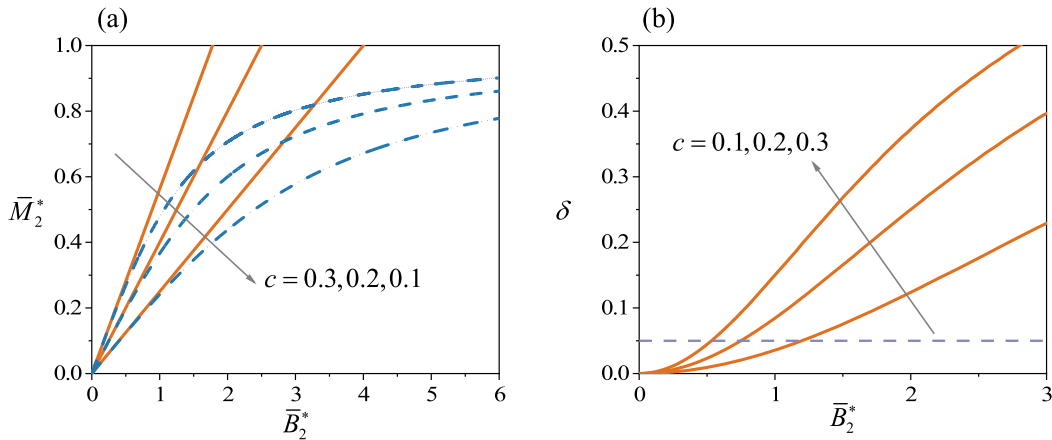


Fig. 5. (a) Dimensionless magnetization $\bar{M}_2^* = \bar{M}_2/\bar{m}_f$ as a function of the dimensionless magnetic induction field $\bar{B}_2^* = \bar{B}_2/\bar{m}_f$ predicted by the Ideal model (solid lines) and the Langevin model (dashed lines) for different particle volume fractions ($c = 0.1, 0.2$, and 0.3). (b) Relative error δ between the dimensionless magnetizations $\bar{M}_2^*$ predicted by the two models as a function of the dimensionless magnetic induction field $\bar{B}_2^*$.

remain nearly identical for $\bar{B}_2^* \lesssim 1.20, 0.74$, and 0.53 (corresponding to dimensional magnetic induction fields of $\bar{B}_2 = \bar{B}_2^* \bar{m}_f \lesssim 2.68, 3.30$, and 3.55 , respectively).

These threshold values $\bar{B}_2^{\text{th}}$ represent the magnetic induction fields at which the relative error δ between the dimensionless magnetization $\bar{M}_2^*$ predicted by the two models reaches approximately 5% (dashed line) in Fig. 5(b). Therefore, for $\bar{B}_2 < \bar{B}_2^{\text{th}}$, the magnetization response can be reasonably approximated as linear and accurately described by the Ideal model, whereas for $\bar{B}_2 > \bar{B}_2^{\text{th}}$, the influence of magnetic saturation becomes non-negligible.

Because the threshold $\bar{B}_2^{\text{th}}$ (or $(\bar{B}_2^*)^{\text{th}}$) depends on the particle volume fraction c , the nonlinear magnetic behavior of the SMA film-substrate system can be effectively tuned by varying c , which is qualitatively consistent with the observations reported by Mukherjee et al. (2020) and Rambausek and Danas (2021). Specifically, the dimensionless threshold $(\bar{B}_2^*)^{\text{th}}$ declines with increasing particle volume fraction c , whereas the dimensional threshold $\bar{B}_2^{\text{th}}$ rises due to the enhanced saturation magnetization $\mu_0 m_f$ associated with larger c . Moreover, an increase in the magnetic induction field leads to larger in-plane stresses in both principal directions required to maintain the plane-strain loading condition.

3.3. Critical bifurcation stretch for a prescribed magnetic induction field

First we consider the scenario where the applied magnetic induction field is fixed at the value $\bar{B}_2$, and the SMA film-substrate system undergoes a homogeneous plane-strain deformation driven by a stress applied along x_1 , while stretch is constrained in the x_3 -direction (i.e., $\lambda_3 = 1$). Here, we take the particle volume fraction to be $c = 0.2$.

Fig. 6(a) shows the bifurcation curves of stretch λ against kH for the SMA film (neo-Hookean Ideal model) subjected to magnetic induction fields $\bar{B}_2 = 0.0, 5.0$ and shear modulus ratios $r = 0.0, 0.1, 0.5$, obtained by solving either of the exact bifurcation Eqs. (12) or (14). When there is no applied magnetic induction field ($\bar{B}_2 = 0.0$) and when the substrate is absent ($r = 0.0$), we recover the results of Cai and Fu (1999) and Wu and Destrade (2021), respectively.

We also observe from Fig. 6(a) that all bifurcation curves, except the $r = 0.0$ curve, start at $\lambda = 0.5437$, the Biot critical stretch of surface instability for an elastic neo-Hookean half-space, which corresponds to $kH = 0$ (vanishing film thickness). They increase to a critical stretch λ_{cr} (the corresponding maximum value) and then decrease monotonically with kH . When $r = 0.0$ (no substrate), the stretch always decreases with kH . All bifurcation curves tend to the same asymptotic limit as $kH \rightarrow \infty$, i.e., as the SMA film becomes a semi-infinite ideal magneto-elastic medium, which is the root of

$$(\lambda^6 + \lambda^4 + 3\lambda^2 - 1)(2 - \chi) = \lambda^2(1 + \lambda^2)\chi^2 \bar{B}_2^2. \quad (29)$$

This equation corresponds to the Biot surface instability criterion in plane strain for an SMA half-space characterized by the neo-Hookean Ideal magneto-elastic model (1), and is in agreement with the more general bifurcation equation established by Wu and Destrade (2021). It yields $\lambda \rightarrow 0.5437, 0.5850$, and 1.2582 when $\bar{B}_2 = 0.0, 2.0$, and 5.0 , respectively.

For a fixed shear modulus ratio, the critical stretch λ_{cr} increases monotonically with increasing applied magnetic induction field. Notably, at sufficiently strong magnetic induction fields, the film-substrate system may undergo wrinkling instability even under extension. Thus, the stable range is determined by the region above the solid line for a fixed $\bar{B}_2$. Furthermore, the bifurcation curve undergoes a nonlinear upward shift when $\bar{B}_2$ is increased (with r fixed) or when r is decreased (with $\bar{B}_2$ fixed), implying that the system is destabilized when the magnetic induction field is increased or when the shear modulus ratio r is decreased. Although the applied $\bar{B}_2$ only exerts a small influence on the critical wavenumber $(kh)_{\text{cr}}$, the latter progressively decreases with a reduction in the shear modulus ratio.

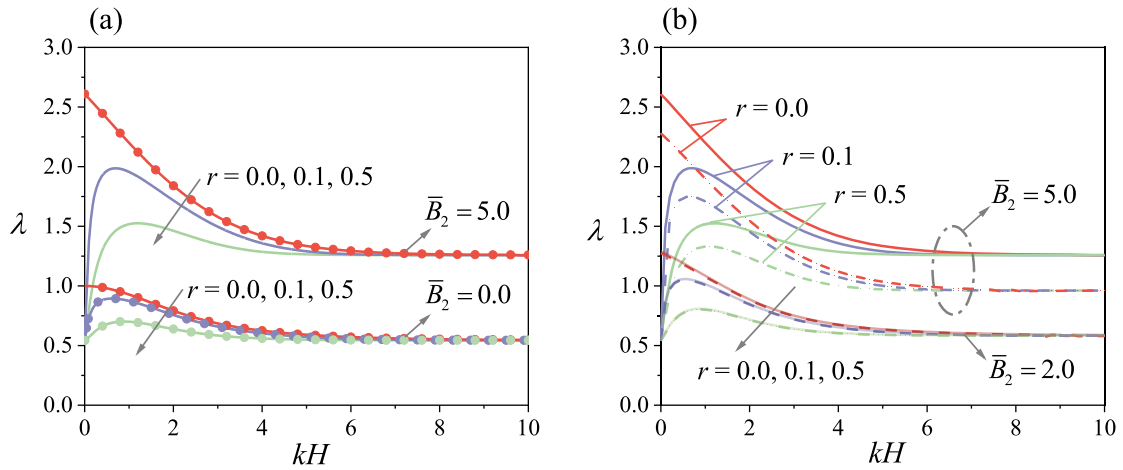


Fig. 6. Bifurcation curves of stretch λ against wavenumber kH for three shear modulus ratios $r = 0.0, 0.1, 0.5$ and magnetic induction fields: (a) $\bar{B}_2 = 0.0, 5.0$ for the Ideal model; (b) $\bar{B}_2 = 2.0, 5.0$ for the Ideal model (solid lines) and the Langevin model (dashed lines). In Fig. 6(a), the dots indicate the results reported in Cai and Fu (1999) when $\bar{B}_2 = 0.0$ and those reported in Wu and Destrade (2021) when $r = 0.0$.

We also compared the differences between using the Ideal model (1) and the Langevin model (2) for the neo-Hookean SMA film in Fig. 6(b), where $\bar{B}_2 = 2.0, 5.0$ and $r = 0.0, 0.1, 0.5$. When $\bar{B}_2 = 5.0$ and $kH \rightarrow \infty$, we have $\lambda \rightarrow 1.2582$ (extension) for the Ideal model and $\lambda \rightarrow 0.9622$ (contraction) for the Langevin model (with the latter evaluated according to the formulation presented in Appendix B of Wu and Destrade (2021)). The critical stretch λ_{cr} obtained by the Ideal model is also notably larger than that of the Langevin model when $\bar{B}_2 = 5.0$. Physically, this indicates that accounting for magnetization saturation under a strong magnetic induction field has a stabilizing effect on the film-substrate system. However, the bifurcation curves calculated by these two material models are almost indistinguishable at $\bar{B}_2 = 2$, since this value falls below the threshold $\bar{B}_2^{th} = 3.30$, where the two constitutive models provide identical static magnetic responses (see the results when $c = 0.2$ in Fig. 5).

3.4. Critical bifurcation magnetic induction field for a fixed pre-stretch

Conversely, by maintaining the pre-stretch at an initially fixed value λ , we can observe surface wrinkling behavior as the applied magnetic induction field $\bar{B}_2$ varies. Fig. 7 illustrates the bifurcation curves of the dimensionless magnetic induction field $\bar{B}_2$ versus the dimensionless wavenumber kH for the two models, covering four specified pre-stretch values ($\lambda = 0.80, 0.90, 1.00, 1.25$) and various shear modulus ratios $r = 0.0, 0.1, 0.5, 1.0$.

We find that, except for the special scenario of $r = 0.0$ (no substrate, where the bifurcation curve increases monotonically), the bifurcation curves of $\bar{B}_2$ versus kH exhibit a minimum, corresponding to the critical magnetic induction field $\bar{B}_{cr}$. Fig. 7 demonstrates that an increase in either the pre-stretch λ or the shear modulus ratio r results in a gradual increase in the critical magnetic induction field $\bar{B}_{cr}$. This trend indicates that the film-substrate system exhibits progressively enhanced stability. Notably, when the pre-stretch is sufficiently contractile ($\lambda < 1$) and the SMA film has a relatively high stiffness (r sufficiently small), surface wrinkling tends to occur under a low magnetic induction field, which is corroborated by the trends in Fig. 7(a) and (b), as well as in Fig. 8. This discovery makes it possible to actively control, in principle, the wrinkles of the system under small magnetic induction fields.

Furthermore, Fig. 7 shows that when the shear modulus ratio r is relatively small, the results of the Langevin model and the Ideal model are very close in the vicinity of the critical magnetic induction field $\bar{B}_{cr}$. However, a significant discrepancy in $\bar{B}_{cr}$ between the two models emerges when r is large. This is because when the SMA film is much stiffer than the substrate (r small), only a relatively weak magnetic induction field is required to trigger wrinkling instability, and the magnetic saturation intensity has virtually no impact. Conversely, when a strong magnetic induction field is needed to trigger wrinkling instability (i.e., when r and λ are relatively large), the bifurcation curves of the Langevin model shift upward compared to those of the Ideal model. Physically, this suggests that under strong magnetic induction fields, the effect of magnetization saturation becomes increasingly significant and contributes to enhanced stability of the film-substrate system, thereby necessitating a higher magnetic induction field to induce wrinkling instability. Furthermore, the increasing prominence of the magnetization saturation effect under strong magnetic induction fields is consistent with the observations reported in Section 3.3.

Fig. 8(a) displays the three-dimensional (3D) surface plot representing the critical magnetic induction field $\bar{B}_{cr}$ under the combined influence of λ and r for the neo-Hookean Ideal magneto-elastic model. Deformations corresponding to points below the surface are stable, while points on the surface correspond to critical states of wrinkling instability (i.e., the critical magnetic induction field $\bar{B}_{cr}$). We can clearly observe that there exist combinations of λ and r for which $\bar{B}_{cr} \rightarrow 0$ (indicated by the spherical symbols in Fig. 8(a)). Furthermore, when the parameter pair (λ, r) lies slightly above the orange demarcation curve $\lambda_{cr}^0 - r_{cr}^0$ in Fig. 8(b), only weak magnetic induction fields are required to effectively regulate the wrinkling behavior of the film-substrate system. Fig. 8(b) shows the corresponding demarcation curve, termed the $\lambda_{cr}^0 - r_{cr}^0$ critical curve, where the critical magnetic induction field is zero.

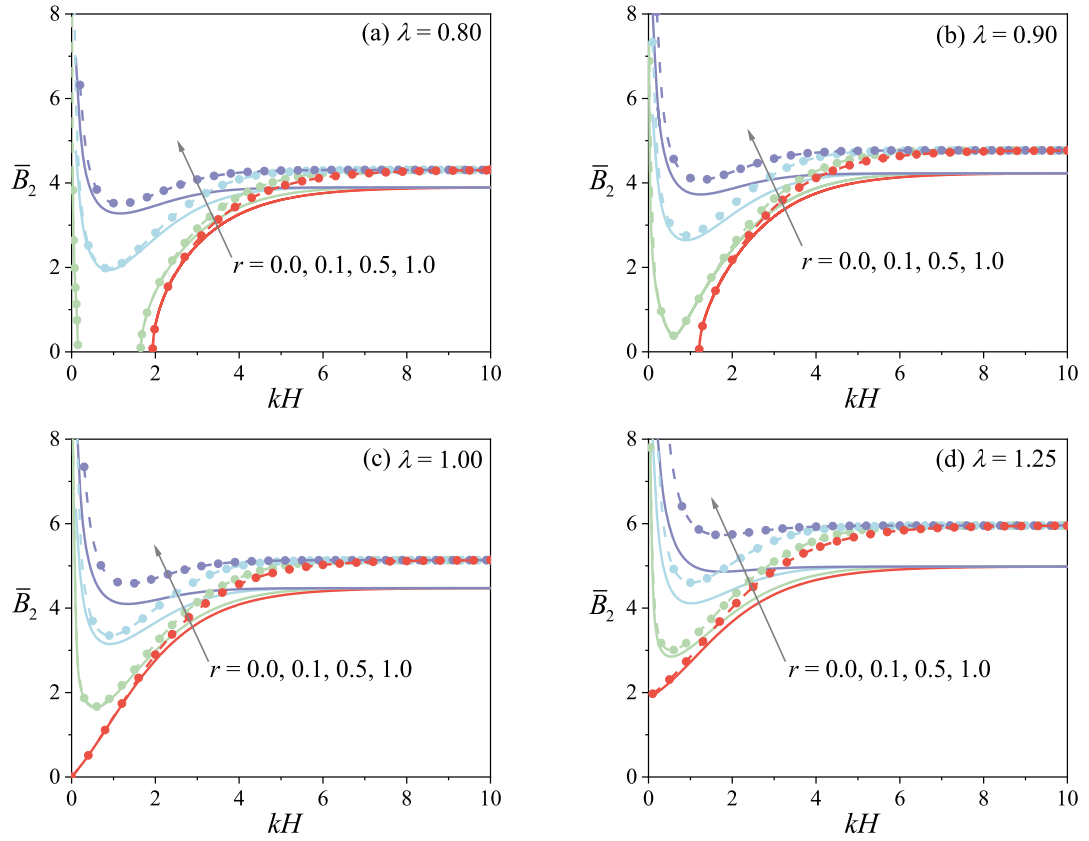


Fig. 7. Bifurcation curves of the dimensionless magnetic induction field $\bar{B}_2$ as a function of kH for different shear modulus ratios $r = 0.0, 0.1, 0.5, 1.0$ and four fixed pre-stretches: (a-b) contractile stretches $\lambda = 0.80$ and $\lambda = 0.90$; (c) no stretch $\lambda = 1.0$; (d) extensile stretch $\lambda = 1.25$. The solid lines show the results of the Ideal model, and the dash-dot lines indicate the results of the Langevin model.

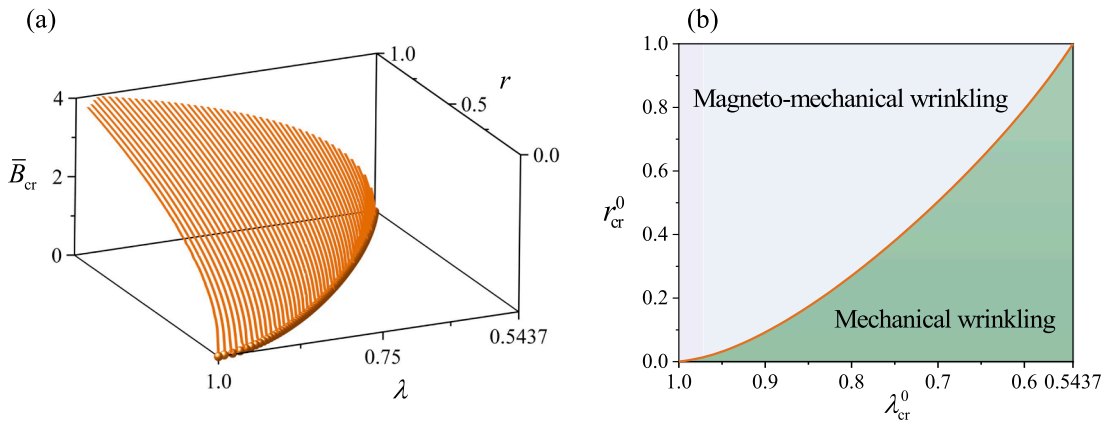


Fig. 8. (a) 3D bifurcation surface illustrating the dependence of the critical magnetic induction field $\bar{B}_{cr}$ on pre-stretch λ and shear modulus ratio r for the Ideal magneto-elastic model; (b) The $\lambda_{cr}^0 - r_{cr}^0$ critical demarcation curve corresponding to the scenario where the critical magnetic induction field is zero (i.e., $\bar{B}_{cr} = 0$, the purely elastic limit), separating the regime of mechanically induced wrinkling from that requiring a finite applied magnetic induction field to initiate wrinkling instability.

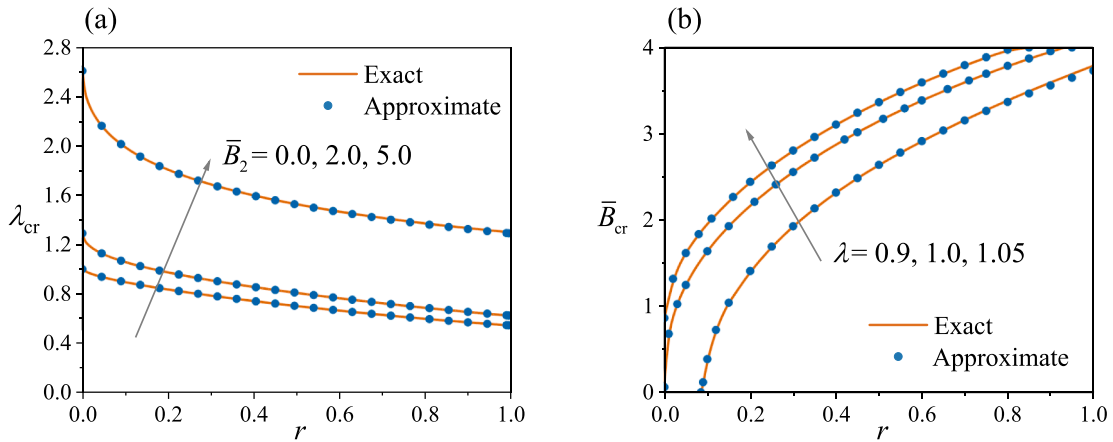


Fig. 9. Curves of (a) critical stretch λ_{cr} as a function of the shear modulus ratio r for different values of $\bar{B}_2 = 0.0, 2.0, 5.0$, and (b) critical magnetic induction field $\bar{B}_{cr}$ versus r for different values of $\lambda = 0.9, 1.0, 1.05$ in the Ideal magneto-elastic model. The solid lines show the results obtained from the exact bifurcation Eq. (12) (or equivalently Eq. (14)), while the symbols indicate the results of the sixth-order approximate bifurcation Eq. (16).

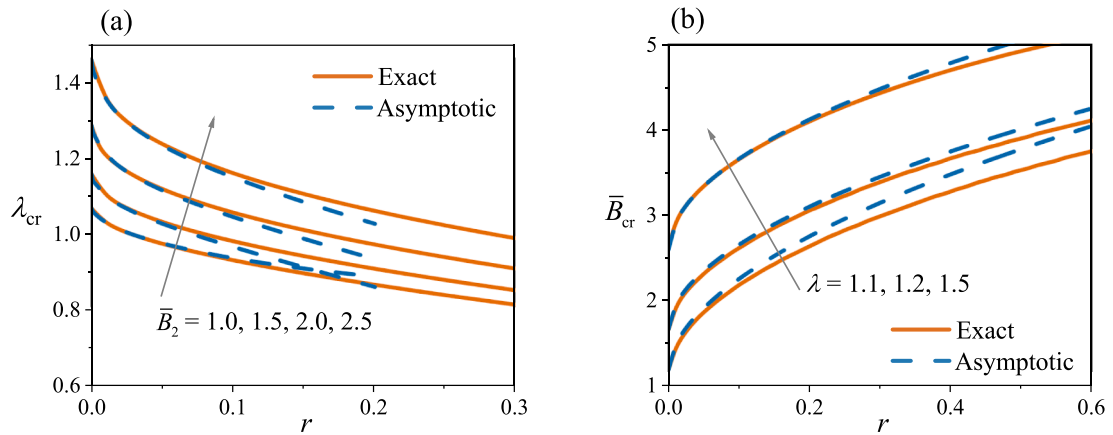


Fig. 10. Variations in critical fields with the shear modulus ratio r for the Ideal magneto-elastic model: (a) critical stretch λ_{cr} for different applied magnetic induction fields $\bar{B}_2 = 1.0, 1.5, 2.0, 2.5$; (b) critical magnetic induction field $\bar{B}_{cr}$ for various pre-stretches $\lambda = 1.1, 1.2, 1.5$. Solid curves: exact solutions given by Eq. (12) (or equivalently Eq. (14)); dashed curves: asymptotic solutions provided by Eqs. (20) and (25).

This purely elastic limit of the critical curve is found by solving either the exact or the sixth-order approximate bifurcation condition at $\bar{B}_2 = 0$. It exhibits an asymptotic behavior described by

$$\lambda_{cr}^0 = 1 - \frac{1}{4}(3r_{cr}^0)^{2/3}, \quad (30)$$

which is consistent with the findings of Cai and Fu (1999) and Wang et al. (2023). Below this demarcation curve, combinations of the pre-stretch λ and the shear modulus ratio r lead to the onset of mechanical wrinkling even in the absence of any applied magnetic induction field, whereas for combinations lying above the demarcation curve, a finite applied magnetic induction field is required to trigger magneto-mechanical wrinkling instability.

3.5. Approximate solution and asymptotic solution

To further investigate the influence of the applied magnetic induction field $\bar{B}_2$ (or pre-stretch λ) on the critical stretch λ_{cr} (or the critical magnetic induction field $\bar{B}_{cr}$) and the corresponding critical wavenumber $(kh)_{cr}$ (or $(kh)^{cr}$), we seek their asymptotic expansions in terms of r and $\bar{B}_2$ (or λ). Fig. 9(a) and (b) show that the curves of the critical stretch λ_{cr} and the critical magnetic induction field $\bar{B}_{cr}$ as functions of r , predicted by the sixth-order approximate solution (16), agrees remarkably well with those obtained from the exact bifurcation Eq. (12) (or equivalently Eq. (14)) over a broad range of r , thereby verifying that it correctly identifies the extrema of the bifurcation curves and laying a solid foundation for the asymptotic analysis of λ_{cr} , $\bar{B}_{cr}$, and $(kh)_{cr}$ (or $(kh)^{cr}$).

The accuracy of the asymptotic expansions in Eqs. (20) and (25) is demonstrated in Fig. 10. Fig. 10(a) presents the variation of the critical stretch λ_{cr} with respect to the shear modulus ratio r , comparing the exact and asymptotic solutions under relatively

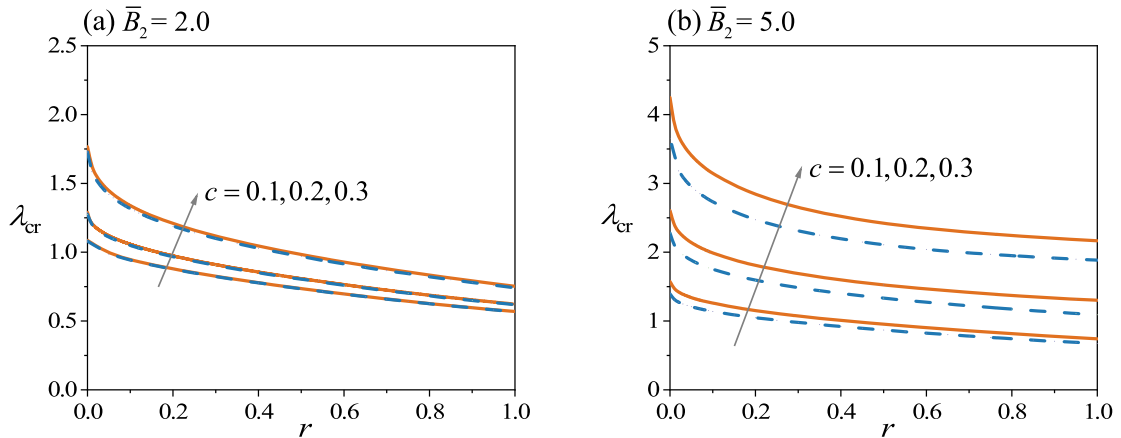


Fig. 11. Critical stretch λ_{cr} as a function of the shear modulus ratio r for three particle volume fractions ($c = 0.1, 0.2, 0.3$) under two different magnetic induction fields: (a) $\bar{B}_2 = 2.0$ and (b) $\bar{B}_2 = 5.0$. Solid and dashed lines represent the predictions of the neo-Hookean Ideal and Langevin models, respectively.

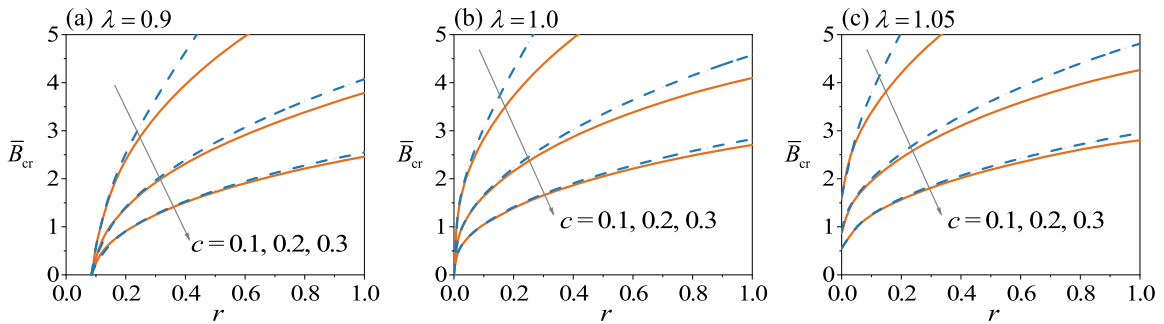


Fig. 12. Critical magnetic induction field $\bar{B}_{cr}$ as a function of the shear modulus ratio r for three particle volume fractions ($c = 0.1, 0.2, 0.3$) under three different pre-stretches: (a) $\lambda = 0.9$, (b) $\lambda = 1.0$, and (c) $\lambda = 1.05$. Solid and dashed lines represent the predictions of the neo-Hookean Ideal and Langevin models, respectively.

strong magnetic induction fields $\bar{B}_2 = 1.0, 1.5, 2.0, 2.5$ for the neo-Hookean Ideal magneto-elastic model, showing excellent agreement for small values of r and good accuracy for moderate values of r . Regarding the dependence of the critical magnetic induction field $\bar{B}_{cr}$ on r under specific pre-stretches $\lambda = 1.1, 1.2, 1.5$, as illustrated in Fig. 10(b), the asymptotic solution shows good agreement with the exact solution over a broad range of r under the pre-extension. Overall, asymptotic expansions of the critical fields provide a fast alternative to solving the exact bifurcation criterion when the soft magnetic-sensitive film is relatively stiffer than the substrate (i.e., for small to moderate values of r).

Previous wrinkling bifurcation studies have been limited to the case of a particle volume fraction of $c = 0.2$. To further elucidate the coupled magneto-mechanical effects arising from the combined influence of the shear modulus ratio r and particle volume fraction c , Fig. 11 investigates the dependence of the critical wrinkling stretch λ_{cr} on r for three different particle volume fractions ($c = 0.1, 0.2, 0.3$) under two applied magnetic induction fields ($\bar{B}_2 = 2.0, 5.0$). The solid and dashed lines denote the predictions from the Ideal model and the Langevin model with magnetic saturation effects included, respectively.

At the lower magnetic induction field of $\bar{B}_2 = 2.0$ in Fig. 11(a), the predictions from the two constitutive models remain nearly identical, consistent with the almost overlapping bifurcation curves observed in Fig. 6. This agreement arises because $\bar{B}_2 = 2.0$ lies below the threshold values $\bar{B}_2^{th}$ for all three particle volume fractions (see Fig. 5). In contrast, at the higher magnetic induction field of $\bar{B}_2 = 5.0$ in Fig. 11(b), a clear deviation emerges between the two model predictions. For a fixed particle volume fraction c and geometric parameter r , the Langevin model predicts a lower critical stretch λ_{cr} than the Ideal model, indicating that magnetic saturation lowers the wrinkling bifurcation threshold. Moreover, increasing the magnetic induction field leads to an increase in the critical stretch. These observations are qualitatively consistent with the trends shown in Fig. 6. For different particle volume fractions c , although the qualitative dependence of λ_{cr} on r remains unchanged, with λ_{cr} decreasing monotonically as r increases, the discrepancy between the two constitutive models becomes more significant with increasing c . This behavior can be attributed to the fact that a larger particle volume fraction not only enhances the relative magnetic susceptibility χ but also strengthens the nonlinear magnetic saturation effect.

To further elucidate the influence of particle volume fraction on the critical magnetic induction field, Fig. 12 presents the variation of $\bar{B}_{cr}$ with the shear modulus ratio r at three prescribed pre-stretches ($\lambda = 0.9, 1.0, 1.05$) for three particle volume fractions ($c = 0.1,$

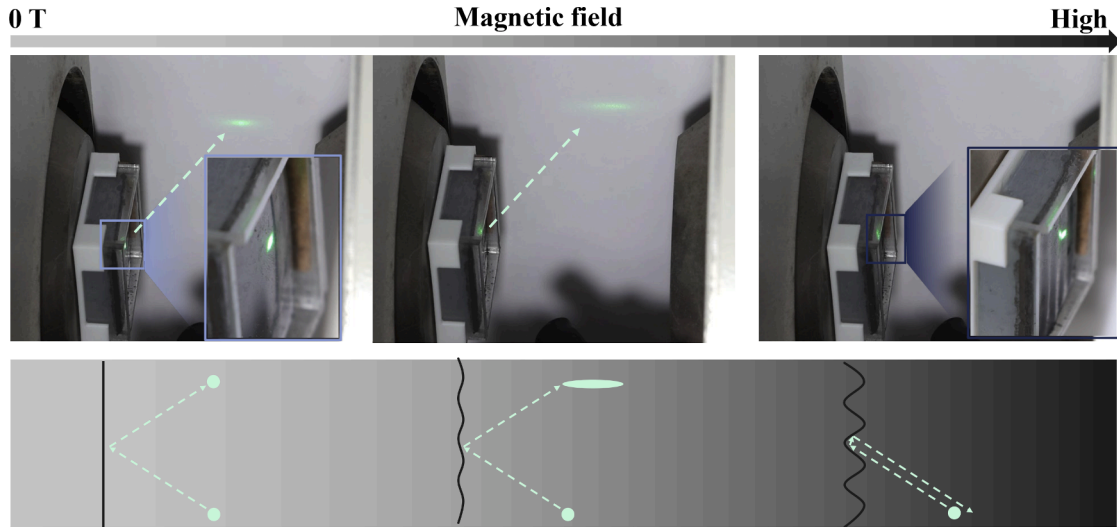


Fig. 13. Magnetic-field-controlled surface wrinkling for optical modulation. A laser beam is reflected from the SMA coating film while the transverse magnetic induction field is increased from low (left) to intermediate (middle), and high (right) values. Top images show the experimental setup and reflected beam: a flat surface at zero field produces a sharp spot, slight wrinkling under an intermediate field broadens the spot, while pronounced wrinkling under a high field suppresses the spot and produces diffuse scattering. Bottom schematics illustrate the corresponding surface profiles and light paths, demonstrating a magnetic-field-driven optical modulation switch with programmable scattering.

0.2, 0.3). For all cases considered, $\bar{B}_{cr}$ increases monotonically with increasing r . Moreover, increasing the pre-stretch from $\lambda = 0.9$ to 1.05 shifts the $\bar{B}_{cr}$ - r curves upward for all particle volume fractions, indicating that a larger pre-stretch enhances the magnetic field required to trigger wrinkling instability. These observations are fully consistent with the trends identified in Fig. 7. In addition, for a given particle volume fraction c , the predictions of the neo-Hookean Ideal and Langevin models remain nearly identical in the low-field regime. Once the critical magnetic induction field enters the regime where magnetic saturation becomes significant, the two predictions gradually diverge, with the discrepancy becoming more pronounced at larger values of r . Furthermore, at a fixed pre-stretch λ , $\bar{B}_{cr}$ decreases with increasing c , reflecting the enhanced magneto-mechanical coupling induced by a higher particle content. Meanwhile, the onset of deviation between the two model predictions occurs at progressively larger r values as c increases. This behavior can be attributed to the fact that increasing c reduces the critical magnetic induction field $\bar{B}_{cr}$, thereby extending the range of r over which the predictions of the two constitutive models remain nearly identical.

Experimentally, we verified that the dimensional critical magnetic induction field B_{cr}^{exp} decreases with increasing particle volume fraction c under different pre-compression levels (see SM-E for details). In Section 3.1, we compared the experimental measurements with theoretical predictions for $c = 0.2$ and demonstrated good agreement between them. However, a reliable quantitative comparison between the theoretical predictions and experimental results for other particle volume fractions would require additional post-bifurcation experiments, together with finite-element-based inverse identification of the SMA film and substrate moduli for each value of c (Psarra et al., 2017). Such an extensive investigation is beyond the scope of the present work.

4. Conclusions

We presented a comprehensive theoretical analysis of the wrinkling instability of an SMA film bonded to semi-infinite hyperelastic substrates under the combined action of a transverse magnetic induction field and in-plane mechanical loading, using two types of neo-Hookean magneto-elastic material models (Ideal model and Langevin model).

By exploiting the advantages of the Stroh formulation and surface impedance method, we managed to derive exact and approximate bifurcation equations that explicitly account for the external incremental magnetic induction field and Maxwell stress contribution in the surrounding vacuum, and subsequently validated their reliability through laboratory experiments. We also deduced explicit asymptotic equations for the critical stretch λ_{cr} , critical magnetic induction field $\bar{B}_{cr}$, and the corresponding critical wavenumbers. Subsequently, we conducted a detailed parameter analysis, including bifurcation diagrams of pre-stretch and magnetic induction fields for various shear modulus ratios and different material models. Finally, we compared the results of the exact solution (12) (or equivalently Eq. (14)), approximate solution (16), and asymptotic solutions (20) and (25). Our main observations are summarized as follows.

First, our findings reveal that the threshold values of the shear modulus ratio r_{cr}^0 and the pre-stretch λ_{cr}^0 , for which $\bar{B}_{cr} \rightarrow 0$ and beyond which magneto-mechanical wrinkling occurs, correspond to purely mechanical instabilities.

Second, the film-substrate system becomes destabilized when the shear modulus ratio decreases, the magnetic induction field increases, or the pre-stretch decreases. In contrast, the saturation magnetization effect enhances the stability of the system, particularly for a high shear modulus ratio, significant pre-extension, or a strong magnetic induction field.

Third, the asymptotic solutions of the critical values agree well with the exact solution when r is small and provide reasonable accuracy for moderate r , in line with the assumptions made, while reducing drastically the computational cost of solving the exact bifurcation equation.

Fourth, increasing the particle volume fraction strengthens the magneto-mechanical coupling within the SMA film, thereby reducing the resistance of the film-substrate system to wrinkling instability. This trend is consistently captured by both the theoretical predictions and experimental observations.

Finally, our results suggest a route to magnetically tunable optical surfaces. Existing wrinkle-based optical control has so far relied mainly on mechanical, thermal, or electric-field loading. While programmable switching between specular and diffuse reflection is well documented in voltage-driven dielectric elastomers (Shian and Clarke, 2016; Chen et al., 2019; Shrestha et al., 2021), the present proof-of-concept experiment demonstrates that similar optical switching can be achieved contactlessly through magnetically induced wrinkling in an SMA film-substrate system. Fig. 13 illustrates this electrode-free, magnetic-field-driven optical modulation switch. In the flat state (in the absence of an applied magnetic induction field) the reflected spot is sharp. Then slight wrinkling broadens the spot, and pronounced wrinkling (under a strong magnetic induction field) suppresses the spot so that only diffuse scattering remains on the coating surface. While quantitative prediction of the post-buckling optical scattering (including finite-amplitude morphology, curvature localization, and inter-domain magnetic interactions, etc.) is beyond the scope of the linearized onset theory presented here, this demonstration confirms that the theoretical instability thresholds can serve as a foundational baseline for designing magnetically addressable, electrode-free optical elements and related devices.

CRedit authorship contribution statement

Linghao Kong: Writing – original draft, Methodology, Formal analysis, Conceptualization; **Wei Jiang:** Writing – review & editing, Visualization, Software; **Congshan Liu:** Writing – review & editing, Visualization, Funding acquisition; **Wei Qiu Chen:** Writing – review & editing, Supervision, Funding acquisition, Conceptualization; **Michel Destrade:** Writing – review & editing, Writing – original draft, Validation, Supervision, Formal analysis, Conceptualization; **Bin Wu:** Writing – review & editing, Writing – original draft, Visualization, Supervision, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

Supplementary material

Supplementary material associated with this article can be found in the online version at [10.1016/j.jmps.2026.106833](https://doi.org/10.1016/j.jmps.2026.106833).

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