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Your Bat-Cape is Ready, Mr. Wayne (But It Might Electrocute You)

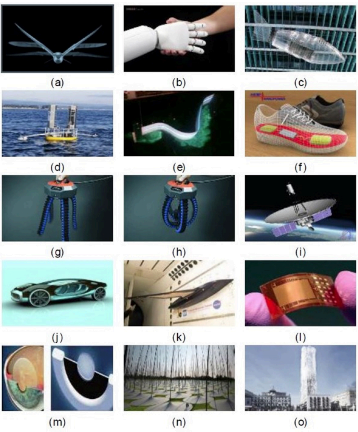
A New Mathematical Formula on Soft 'Dielectric' Membranes

By Giuseppe Zurlo and Michel Destrade

In *Batman Begins*, Lucius Fox (played by Morgan Freeman) offers Batman (played by Christian Bale) an unconventional shield made from soft fabric that becomes rigid when electricity runs through it.

In the 2005 *Batman Begins* movie, Bruce Wayne (portrayed by Christian Bale) is in search of new weapons for his Batman alter ego. Lucius Fox, the effortlessly handsome CEO of Wayne Enterprises (played by Morgan Freeman), obliges him with an unconventional shield made from soft fabric — the "memory cloth." It becomes instantly rigid when electricity runs through it; thus, the Bat-Cape is born!

We thought of that scene when we finally managed to model the catastrophic thinning of dielectric elastomers [1]. Indeed, in the 13 years since the movie's release, the science of electro-active elastomers has progressed tremendously. These materials are soft solids that can deform not only under the action of mechanical forces and moments, but also when subject to an electric current. Their potential for applications is enormous. For example, they can be used to create soft robots, smart clothes, or artificial muscles, which contract or expand at will via electricity. Conversely, they can be used as harvesters of electrical energy; imagine that an electro-active sheet is placed in your shoe under the sole of your foot. Your heel compresses it with each step and the sheet expands, creating a small electrical current. We can then easily think of many low-energy devices (cell phone, anyone?) that could benefit from a small regular electrical boost.



Recent applications of electro-active polymers. **(a)** Dielectric elastomer actuated flapping wing. **(b)** Dielectric elastomer actuated hand. **(c)** Dielectric elastomer actuated biomimetic airship. **(d)** Dielectric elastomer ocean power generator. **(e)** Piezoelectric eel. **(f)** Piezoelectric shoe. **(g)** and **(h)** Electrically actuated device. **(i)** Piezoelectric actuated space antenna. **(j)** Piezoelectric technology incorporated in automobile. **(k)** Boeing MFX-1: morphing aircraft test at Langley NASA. **(l)** Piezoelectric nano-harvester. **(m)** Dielectric elastomer biomimetic lens. **(n)** Piezoelectric wind farm. **(o)** Piezoelectric skyscraper. Image courtesy of Rogelio Ortigosa Martínez.

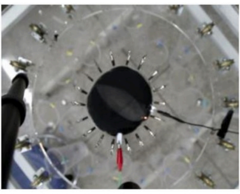
A significant limitation of these membranes is that they must be put under a very large voltage to expand significantly. To achieve this high voltage, experimentalists reduce the thickness of the membrane as much as possible (because the voltage between the two faces of the membrane is inversely proportional to its thickness). Hence, most prototypes are at most 1 or 2 mm thick, and sometimes as thin as 0.1 mm (standard household foil is typically 0.016 mm). But another problem arises: as the membrane expands in its plane under the action of the electric field, its thickness decreases rapidly, ultimately causing a short-circuit and catastrophic breakdown. In the following animation, we show experiments conducted by our engineering colleagues in the SoftMachine Lab at Xi'an Jiaotong University, China. The clip demonstrates that a disc of dielectric membranes can deform tremendously under a strong voltage (its diameter can increase fivefold), but will eventually break very suddenly.

Catastrophic thinning of dielectric elastomers. Animation credit: Giuseppe Zurlo and Michel Destrade.

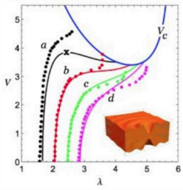
The main challenge is thus to predict how much voltage a given electro-active membrane can sustain, if only to provide safety guidelines. Working with collaborators at Politecnico di Bari in Italy, we created the following simple formula to link the physical properties of the membrane to the breakdown amount of stretch:

$$V_c = \frac{2}{\sqrt{3}}\sqrt{\frac{W\prime}{\mu}}\min\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right),$$

where V_c is a non-dimensional measure of the critical voltage, $W\prime$ is the derivative of the elastic energy of the material, μ is its initial shear modulus (a measure of how much it can resist shearing forces), and λ_1, λ_2 are the membranes' lateral stretches. We found that our formula yielded a remarkable agreement with experimental results. We also revealed that a catastrophic deformation occurs upon reaching this critical voltage; as shown by energy arguments, this deformation cannot be reversed. We also found the membranes' shape, although of course it will be nigh to impossible to observe them, as the corresponding thinning occurs in the blink of an eye.



We are preparing an animation of how this membrane stretches when a voltage is applied.



The point plots are the experimental loading curves (voltage Vs stretch) for different pre-stretched discs. They stop when the membranes break, a phenomenon captured quite accurately by our blue curve.

Previous predictions of critical voltages were successful in some cases, but none were as universal and unifying as the predictions resulting from our formula. To derive it, we performed expansions of the membrane's total energy in powers of (small thickness) up to third order, as opposed to first order in the literature. This allowed us to identify the pinching deformation of the membrane at the critical threshold as completely unstable (hence the "catastrophic" adjective).

We are now refining the formula to include a wider class of electro-active polymers. We are also working in close collaboration with our friends at Xi'an Jiaotong to perform more amazing experiments that will test those materials to their limits. The Bat-Cape might be in sight, but it may give its owner a nasty electric shock upon eventual activation!

References

[1] Zurlo, G., Destrade, M., DeTommasi, D., & Puglisi G. (2017). Catastrophic Thinning of Dielectric Elastomers. *Phys. Rev. Lett.*, *118*, 078001.

Giuseppe Zurlo is a professor in the School of Mathematics, Statistics and Applied Mathematics at the National University of Ireland, Galway (NUI, Galway).

Michel Destrade is chair of applied mathematics in the School of Mathematics, Statistics and Applied Mathematics at NUI, Galway and a visiting professor of mechanical engineering at University College Dublin.