How many presents did my true love really send to me?

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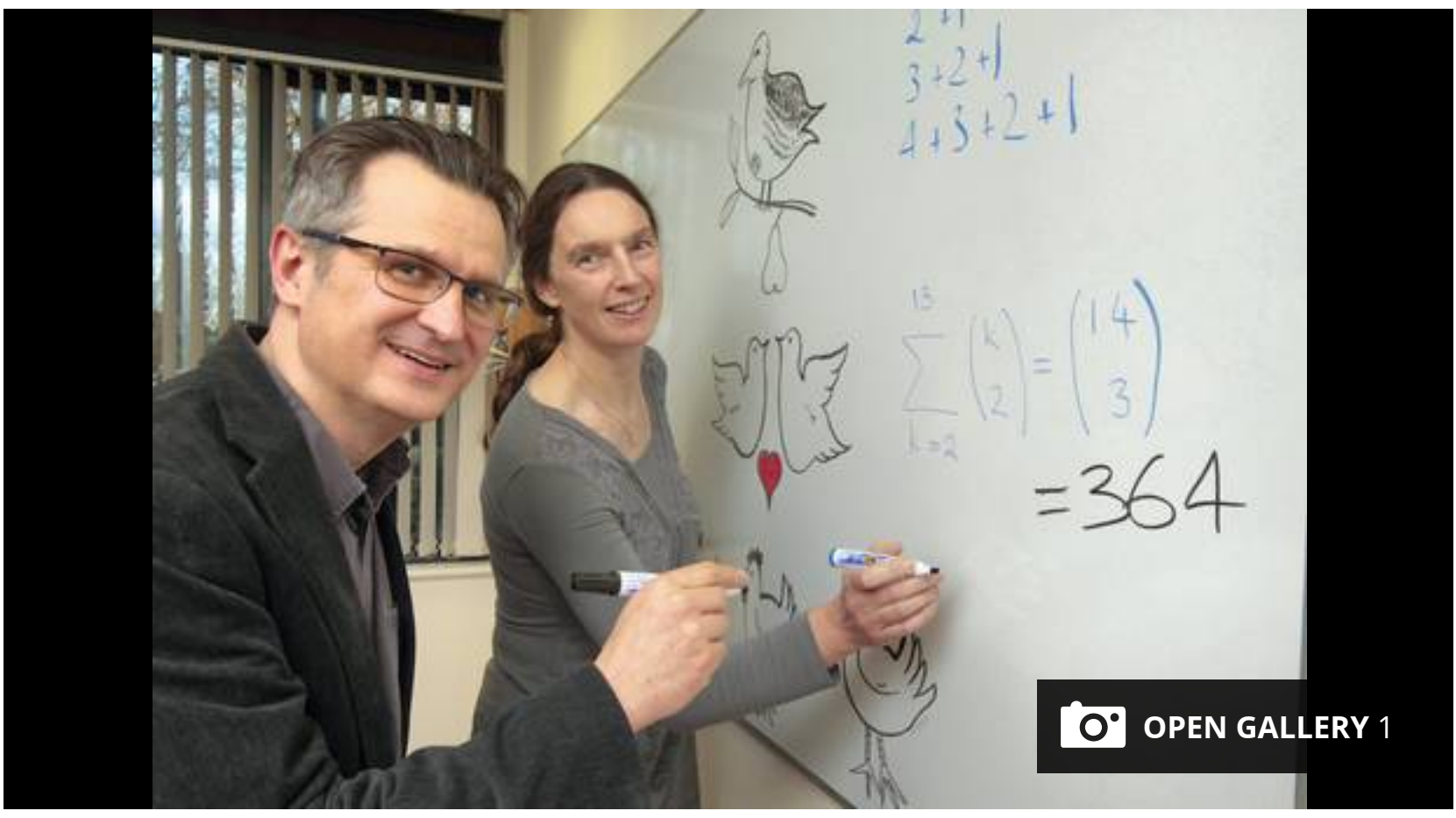

Dr Rachel Quinlan and Prof Michel Destrade of NUI Galway school of Mathematics.

Over the 12 days of Christmas, how many gifts did my true love send to me? The obvious answer might seem to be 1+2+3+4+5+6+7+8+9+10+11+12, but inspection of the lyrics reveals that this vastly underestimates the true total. According to our calculations, I would need to give my true love a daily present for a whole year in order to surpass his generosity.

On the first day of Christmas my true love sent to me a partridge in a pear tree - so that's one present for me. On the second day, this was followed by two turtle doves and another partridge. So that's 1+2=3 presents that day and a total of 1+3=4 presents so far. On each following day, all of the previous day's gifts were repeated and more added to the list, until Day 12.

Finding a neat way to calculate the number of presents received on a particular day is an interesting mathematical challenge. A famous story recounts its solution in 1784 by the great German mathematician Carl Friedrich Gauss. The 7-year-old Gauss reputedly amazed his teacher by adding up all the whole numbers from one to 100 in a matter of seconds.

Using the Gauss approach, the number of presents received on Day 6, or example, is calculated as one half of the sum 1-6+6-1. So, to calculate my gift haul on Day 6, we need to add 1+2+3+4+5+6. Let's write this sum twice, in reverse order the second time: (1+2+3+4+5+6)+(6+5+4+3+2+1).

We find that by pairing numbers in each sequence, 1 and 6, 2 and 5, 3 and 4, etc, we end up with 6 pairs of numbers that add up to 7. So twice the sum is 6x7=42 and the sum itself is 21. That's exactly how Gauss astounded his teacher with the answer "5,050", which is half of 100x101.

On the first day of Christmas my true love sent one present, which is half of 1x2; on the second day he sent 3 presents, which is half of 2x3; on the third day, he sent half of 3x4, which is 6, etc.; until Day 12, when he sent half of 12x13, which is 78 presents.

The total number of presents over the 12 days will thus be the sum of 1+3+6+10+15+21+28+36+45+55+66+78, which is 364, one item short of one present per day for a whole (non-leap) year.

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* See page 18.

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